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Interplay between consensus and coherence in a model of interacting opinions

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HIGHLIGHTS

- Opinion formation model based on multi-layer extension of the Ising model.
- Effect on consensus of the interplay between social pressure and internal coherence.
- Action of external fields allows states of partial consensus and full coherence.
- Qualitative behavior preserved under thermal noise below a critical temperature.

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ABSTRACT

The formation of agents' opinions in a social system is the result of an intricate equilibrium among several driving forces. On the one hand, the social pressure exerted by peers favors the emergence of local consensus. On the other hand, the concurrent participation of agents to discussions on different topics induces each agent to develop a coherent set of opinions across all the topics in which he/she is active. Moreover, the pervasive action of external stimuli, such as mass media, pulls the entire population towards a specific configuration of opinions on different topics. Here we propose a model in which agents with interrelated opinions, interacting on several layers representing different topics, tend to spread their own ideas to their neighborhood, strive to maintain internal coherence, due to the fact that each agent identifies meaningful relationships among its opinions on the different topics, and are at the same time subject to external fields, resembling the pressure of mass media. We show that the presence of heterogeneity in the internal coupling assigned by agents to their different opinions allows to obtain states with mixed levels of consensus, still ensuring that all the agents attain a coherent set of opinions. Furthermore, we show that all the observed features of the model are preserved in the presence of thermal noise up to a critical temperature, after which global consensus is no longer attainable. This suggests the relevance of our results for real social systems, where noise is inevitably present in the form of information uncertainty and misunderstandings. The model also demonstrates how mass media can be effectively used to favor the propagation of a chosen set of opinions, thus polarizing the consensus of an entire population.

1. Introduction

The increasing availability of data sets about social relationships, such as friendship, collaboration, competition, and opinion formation, has recently spurred a renewed interest for the basic mechanisms underpinning human dynamics [1]. Aside with the classical studies in social sciences and social network analysis [2–4], some interesting contributions to the understanding of social dynamics have lately come from statistical physics [5], which has brought in the field new tools and analytical methods to study systems consisting of many interacting agents. In such wider context, much effort has been devoted to the study of the dynamics responsible for opinion formation in populations of interacting agents, and in particular to a more in-depth understanding of the elementary mechanisms allowing the emergence of global consensus and of the role of endogenous and exogenous driving forces, including social pressure and mass media. As a result of this investigation, a plethora of models of opinion formation have been proposed and studied [6–13].

Although the majority of those models originally made the simplifying assumption of considering homogeneous interaction patterns (basically, regular lattices), the rise of network science

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[14–17] provided the tools to overcome this limitation, featuring more realistic interaction patterns. More recently, also the role of mass media in the formation of global consensus has attracted a lot of interest [18–23].

An aspect of social relationships that has been mostly discarded in the study of the emergence of consensus is the fact that agents usually interact in a variety of different contexts, making the interaction pattern effectively multilayered and multi-faceted. As a matter of fact, the urge to maintain a certain level of coherence among opinions on different but related subjects might actually play a crucial role in determining the reaction of each agent to external pressure and in facilitating (or hindering) the emergence of global consensus. Moreover, the balance between the internal tendency towards coherence and the necessity to adequately respond to social pressure is naturally dependent on each person's attitude, thus implying a certain level of heterogeneity. Some individuals may be more prone to align more closely to the opinions of their neighbors in each of the different contexts where they interact, putting little or no importance to the overall coherence of their profile. On the contrary, some other agents may indeed be more reluctant to change their opinion on a topic, in spite of being urged by other individuals or media, if such a change results in a contradiction with another of their opinions on a different but related subject.

In this paper we propose a model of opinion formation that takes into account (i) the concurrent participation of agents to distinct yet connected interaction levels (representing discussion topics or social spheres), (ii) the presence of social pressure and (iii) the exogenous action of mass media. Our analysis can be naturally cast in the framework of multiplex networks [24–27], which has recently proven successful for a more realistic modeling of different social dynamics [28,29,21,30–32]. According to this framework, agents are represented by nodes connected by links of different nature, where links of the same kind belong to the same layer of the system. Each layer thus represents the interaction pattern of individuals discussing a given topic. Different layers are in general endowed with different topologies, to mimic multi-layer real-world social systems where distinct interaction patterns are present at different levels. Peer social pressure occurs on each topic through intra-layer links. The opinions of an individual on the different topics are also driven towards a specific state by the tension towards internal agent's coherence, represented by a preferred configuration of opinions on different topics. Mass media are introduced as fields acting uniformly on all the agents at the level of each single topic.

The resulting model is a natural extension of the traditional Ising model of magnetic interaction [33] and of more recent variations introduced to take into account the effect of external forces on the emergence of consensus [34], in the spirit of less and more recent works connecting statistical mechanics of disordered systems and opinion dynamics [35–37]. The key ingredient of heterogeneous distributed couplings between opinions lead to interesting equilibrium states, where agents can remain fully coherent while a variable level of global consensus is attained, depending on the strength of the pressure exerted by mass media. This clearly resembles the dynamics observed in real societies, thereby supporting the relevance of our approach.

2. Model

We consider a population of N individuals interacting through M different layers, representing different topics or subjects. The network of each layer $\alpha = 1, \ldots, M$ represents the pattern of interactions among agents on a specific topic, which is in general distinct from those of the other layers, and is encoded by the adjacency matrix $A^{[\alpha]} = \{a_{ij}^{[\alpha]}\}_{i,j=1,\ldots,N}$, whose element $a_{ij}^{[\alpha]} = 1$ only if agent i and agent j are neighbors on layer α, and equal to zero otherwise. The structure of the overall interaction pattern is thus concisely represented by the vector of adjacency matrices $\mathcal{A} = \{A^{[1]}, \ldots, A^{[M]}\}$, where all the matrices $A^{[\alpha]}$ are in general distinct. Each agent $i = 1, \ldots, N$ expresses a binary opinion $s_i^{[\alpha]} = \pm 1$ on each subject $\alpha = 1, \ldots, M$. An example with $M = 2$ is shown in Fig. 1(a), where upwards and downwards spins represent the two possible values of $s_i^{[\alpha]}$. We assume that agent opinions evolve over time due to two concurrent mechanisms. On the one hand, agents are subject to social pressure from their peers on each layer (denoted by the red and blue links in Fig. 1), so that the opinion of agent i on node α will tend to remain aligned with the opinions of its neighbors on the same layer. This mechanism, based on the elimination of conflicting opinions on a microscopic scale, has been widely observed in many real-world social systems [38], and is responsible for the attainment of local consensus on each layer. On the other hand, we assume that the opinions of agent i at the different layers are not independent from each other but are instead interacting, so that for each agent there exists a preferred configuration of opinions at the different layers which is considered *coherent*. For instance, the political orientation of a person is often related to his/her ideas about economy and welfare, so that the emergence of consensus with its neighbors on one subject should remain coherent with its current opinions on the other layers. Moreover, we imagine that agents are exposed, on each layer, to the action of mass-media, a mean-field external force which preferentially drives their opinions towards either $+1$ or -1.

We formalize the interplay of these concurrent dynamics by defining the functional:

$$f_i^{[\alpha]} = J \sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]} + h^{[\alpha]} + \gamma \frac{\chi_i}{J} \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^{M} s_i^{[\beta]} \tag{1}$$

for each agent i and each topic α. The first sum on the rhs of Eq. (1) represents the social pressure exerted on i by its neighbors on layer α, and is weighted by the coefficient J, which models its intrinsic permeability to social pressure. The variables $h^{[\alpha]}$ represent the external effect of mass-media on the formation of agents' opinions, which are considered in this case as a mean-field force acting homogeneously on all the agents of a layer. Finally, the second sum represents the tendency of agent i towards internal coherence, where the global parameter γ sets the relative importance of internal coherence and social pressure. Specifically, when $\gamma \simeq 0$ the opinions of the agents are mainly driven by peer and external pressure, whereas when $\gamma \to \infty$ they are determined by the internal coherence, such that coherent configurations are strongly favored.

This setup is depicted in Fig. 1(b) for the case $M = 2$, where links with different colors indicate the connections of an agent at the two layers. In practice, J is the strength of the interaction of each agent with its neighbors, while χ_i determines the importance (and sign) of internal agent coherence. In this case, as shown in Fig. 1(c), the preferred configuration of agent's i spins is concordant if $\chi_i > 0$ and discordant if $\chi_i < 0$. We notice that the actual value of χ_i, which in the following always lie in the interval $[-1, 1]$, is a measure of how much agent i is flexible towards a change of one of its opinions, eventually leading to configurations which do not agree with what it would consider a coherent configuration of its spins. In other words, agents for which $|\chi_i| \simeq 0$ assign less importance to internal coherence and more relevance to social pressure, while the opposite happens when $|\chi_i| \simeq 1$.

In our model, the opinions of each agent evolve towards configurations which maximize the function $F_i^{[\alpha]} = s_i^{[\alpha]} f_i^{[\alpha]}$, in order to attain, at the same time, internal coherence and local