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Contact-based model for strategy updating and evolution of cooperation

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HIGHLIGHTS

- A different way of establishing strategy update is proposed.
- Strategy update depends on the switching probability between strategies.
- The conditions fostering the coexistence of strategies can be calculated.
- Consulting more agents for strategy update promotes the coexistence of strategies.

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ABSTRACT

To establish an available model for the astoundingly strategy decision process of players is not easy, sparking heated debate about the related strategy updating rules is intriguing. Models for evolutionary games have traditionally assumed that players imitate their successful partners by the comparison of respective payoffs, raising the question of what happens if the game information is not easily available. Focusing on this yet-unsolved case, the motivation behind the work presented here is to establish a novel model for the updating of states in a spatial population, by detouring the required payoffs in previous models and considering much more players' contact patterns. It can be handy and understandable to employ switching probabilities for determining the microscopic dynamics of strategy evolution. Our results illuminate the conditions under which the steady coexistence of competing strategies is possible. These findings reveal that the evolutionary fate of the coexisting strategies can be calculated analytically, and provide novel hints for the resolution of cooperative dilemmas in a competitive context. We hope that our results have disclosed new explanations about the survival and coexistence of competing strategies in structured populations.

1. Introduction

Despite its ubiquity in nature and human societies, the survival of cooperative behaviours among selfish individuals when defection is the most advantageous strategy is still puzzling [1–5]. Hence, it has long been a subject of fascination and a considerable effort has been made to address this puzzle [6–8]. Among a large amount of solutions, the study of complex networks has provided new grounds to the understanding of cooperative behaviours in the framework of evolutionary game theory [9–15]. The integration of the microscopic patterns of interactions among players becomes a central topic to study population dynamics in paradigmatic scenarios. Effects of network topologies, or equivalently population structures, on the evolutionary processes have been discussed intensively, and with the rapid development of complex network theory, these effects are gradually unravelled [16–21]. Accordingly, there exists a large evolutionary game literature collectively exploring effects of population structure on evolutionary outcomes. And, complex network theory has paved the way for exploring many real-world large-scale networks, and describing and understanding various processes that evolve in typical such networks [22–30], whose structural nature are also allowed to vary.

Evolutionary game dynamics generally involve how players update their strategies as time evolves. Several strategy update rules are customary in evolutionary game theory. Many evolutionary game theoretic studies have extensively considered the cases of the updating rules based on replication or imitation, which play a crucial role in enabling the evolution of strategies [31–34]. The essence of replication rules is that a strategy with better performances has a higher replication rate. Imitation rules are also a

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broad and relevant class of update rules where a focal individual and a model are randomly selected from the population and a probabilistic comparison of their respective fitness determines whether the focal individual switches strategy and adopts the strategy of the model [35–38].

Interesting and often creative, though the mentioned strategy update rules have been, they are hypotheses beyond practice to a certain extent. Generally, to apply such update rules, there is still too much emphasis placed on the exact magnitudes of the payoffs of all the referred players. However, inferring payoffs may not be as easy as it is often assumed: individuals' bounded rationality implies their limited cognition and decision-making capabilities [39–41]; and, computations might be cognitively expensive and thus unfavourable. One of the most likely source of the problem is the difficulty inherent in measuring the required payoffs. In addition, when a player interacts with more than one players, the problem of how she adjusts her strategy becomes complicated. For that matter, theoretical evidence is sparse in regard to modelling the specific strategy updating process and the interplay among the linked partners and their effects on the evolution of cooperation. Therefore, very little is known about the evolutionary conditions under which strategies can evolve, however, it is precisely these conditions which sway self-interested agents towards cooperation or defection. Simply stated, the information acquisition ability and the corresponding results probably vary among agents. Thus, the boundedness owned by the general strategy update rules help motivate the encouraging study on more realistic strategy update rules. Also, how cognitive processes with limited information take place in game playing needs to be investigated as the theoretical evidence is lagging behind of insights of natural observations. Based on the intuition gained from these discussions, the intent of this work is to tackling the above-mentioned question by proposing a general contact-based model.

Our previous work [42] represents a primitive attempt to introduce a new strategy changing rule, an intriguing feature of which is the absence of usually required payoff information. Our interest is focused primarily on a novel approach, bypassing the requirement for explicit knowledge of the exact payoffs, by encoding the payoffs into the willingness of any player to switch from her current strategy to the competing (cooperation or defection) one. We provide a framework to investigate the evolution of how players in large structured populations choose from two competing strategies after repeatedly playing games with their opponents. The upshot that inspires us discover new mechanisms is based on players' contact patterns and the switching probability between strategies. Theoretical computations and numerical simulations collectively show that the evolutionary dynamics are intrinsically regulated by the contact relationships specified by the network topologies of the populations.

Clearly these assumptions may form a starting point for modelling and compromise the ability of the resulting models to form realistic representations of many gaming scenarios. In our previous work [42], the general model for the updating of states in a network allows us to effectively derive the conditions under which the steady coexistence of strategies is feasible. Theoretical models there, however, still have some limitations concerning the conditions under which the strategy switching happens. Perhaps the unnoticed issue is that strategy revisions can also occur in a pair of agents adopting the same strategy, and it is exactly our concern here. Actually, it has been noticed that in the real world, an important phenomenon is that players reach a decision on the basis of multiple factors. Thus, it is an interesting problem to establish a general model by relaxing the limitation that strategy switching only occurs in pairs of different strategies, allowing for the possibility of strategy switching at any case. To represent this, we propose a more general model covering a wider range of update situations among interacting players, affording us a much clearer understanding of the real phenomenon in social systems.

The rest of the paper is as follows: in the next section we describe the model in ample detail. Sections 3 and 4 are devoted to the presentation of main findings, whereas in the last section a summary and conclusions can be found.

2. Model setting

Here we consider a network of N ($N \gg 1$) players, labelled by $1, \dots, N$, each of whom has two candidate strategies A and B to play against one another. They actually play with mixed strategies. We thus use p_i to denote the probability that player i chooses the strategy A, and obviously she adopts strategy B with probability $1 - p_i$.

More specifically, the network topology determines completely who meets whom, and we use the N-by-N adjacency matrix $(a_{ij})_{N\times N}$ to describe the players' interaction pattern, where $a_{ij} = 1$ if and only if players i and j play against each other in the network and $a_{ij} = 0$ otherwise. It follows immediately from this definition that the adjacency matrix of a network is symmetric, e.g. $a_{ij} = a_{ji}$.

As pointed above, we encode the payoffs associated with a game between two players into the willingness of a player to shift her current strategy to the other one after playing with her opponent; more specifically, we denote by $u_{B\to A|A}$ the willingness that a B-player shifts her strategy from B to A in the presence of her partner A, and correspondingly $u_{A\to B|B}$ the willingness that an A-player adopts strategy B after playing with a B-player. Similarly, $u_{A\to B|A}$ denotes the probability that an A-player will switch to the alternative strategy B when she plays with an A-player. $u_{B\to A|B}$ denotes the probability that an agent using strategy B will switch to another selectable strategy A when this agent connects with a B-player. Moreover, it is worth emphasizing that all agents simultaneously update their strategy in such a way.

Having defined our strategic context, we now turn to the dynamics. A major point to note is that here players interact with neighbours and p_i changes with time. We aim to study in this N-player network, whether the competing strategies (A and B) may coexist in the long run; in addition, if, to the contrary of intuition, the answer to this question is yes, how they coexist and the related factors. This issue receives more attention in our study, and we first look into the discrete-time model for the evolution of the probability that any player i plays with strategy A, as described by

$$p_i(t+1) = p_i(t)[1 - U^i_{A\to B}(t)] + [1 - p_i(t)]U^i_{B\to A}(t), \qquad (1)$$

where $U^i_{A\to B}(t)$ is the tendency that player i's strategy switches from A to B, and similarly $U^i_{B\to A}(t)$ is the tendency that player i's strategy changes from B to A. The above equation assumes that the update rule is a memoryless Markov process, then for all $i = 1, \dots, N$, $U^i_{A\to B}(t)$ and $U^i_{B\to A}(t)$ are crucial factors.

Put simply, these assumptions mentioned above make the basket of the scenarios, when the strategy update occurs, bigger than the previous version. Clearly, the above equation is similar with Eq. (4) in our earlier theoretical study [42], by sharing the main idea that strategy updating is driven by the switching probabilities. However, it is plausible to improve the original model by assuming the strategy switching occurs between any pair of players (AA, AB, BA or BB) in structured populations and not just the competing ones (AB or BA) in our previous work.

3. Evolutionary dynamic results

Before presenting our results in detail, we define our formal framework and then discuss the validity of our assumptions about