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Cascades in interdependent flow networks

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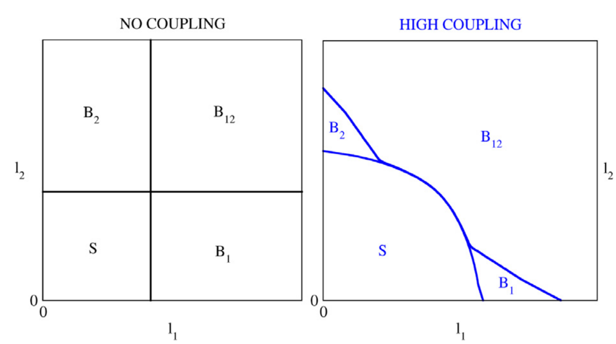
HIGHLIGHTS

- We introduce a new analytical model of coupled cascades in flow networks.
- We find that increasing coupling enhances the safety of the coupled systems.
- However, increasing coupling makes the systems more likely to fail together.

GRAPHICAL ABSTRACT



Effects of coupling networks on cascading failures: while coupling increases the area **S** where no system wide cascade occur, it also decreases the area $\mathbf{B_1}$, $\mathbf{B_2}$ where only one system fails and increases the region $\mathbf{B_{12}}$ where both system fail together. Hence, the two systems together behave more like a safer single system that however fails altogether.

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ABSTRACT

In this manuscript, we investigate the abrupt breakdown behavior of coupled distribution grids under load growth. This scenario mimics the ever-increasing customer demand and the foreseen introduction of energy hubs interconnecting the different energy vectors. We extend an analytical model of cascading behavior due to line overloads to the case of interdependent networks and find evidence of first order transitions due to the long-range nature of the flows. Our results indicate that the foreseen increase in the couplings between the grids has two competing effects: on the one hand, it increases the safety region where grids can operate without withstanding systemic failures; on the other hand, it increases the possibility of a joint systems' failure.

1. Introduction

Physical Networked Infrastructures (*PNIs*) such as power, gas or water distribution are at the heart of the functioning of our society; they are very well engineered systems designed to be at least $N-1$

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robust—i.e., they should be resilient to the loss of a single component via automatic or human guided interventions. The constantly growing size of *PNI*s has increased the possibility of multiple failures which escape the $N-1$ criteria; however, implementing robustness to *any sequence* of k failures ($N-k$ robustness) requires an exponentially growing effort in means and investments. In general, since *PNI*s can be considered to be aggregations of a large number of simple units, they are expected to exhibit emergent behavior, i.e. they show as a whole additional complexity beyond what is dictated by the simple sum of its parts [1].

A general problem of *PNI*s are cascading failures, i.e. events characterized by the propagation and amplification of a small number of initial failures that, due to non-linearity of the system, assume system-wide extent. This is true even for systems described by linear equations, since most failures (like breaking a pipe or tripping a line) correspond to discontinuous variations of the system parameters, i.e. are a strong non-linear event. This is a typical example of emergent behavior leading to one of the most important challenges in a network-centric word, i.e. systemic risk. An example of systemic risk in *PNI*s are the occurrence of blackout in one of the most developed and sophisticated system, i.e. power networks. It is important to notice that if such large outages were intrinsically due to an emergent behavior of the electric power systems, increasing the accuracy of power systems' simulation would not necessarily lead to better predictions of black-outs.

Power grids can be considered an example of complex networks [2] and hence cascading failures in complex networks [3] is field with important overlaps with system engineering and critical infrastructures protection; however, most of the cascading models are based on local rules that are not appropriate to describe systems like power grids [4] that, due to long range interactions, require a different approach [5,6].

Another important issue is increasing interdependence among critical infrastructures [7]; seminal papers have pointed out the possibility of the occurrence of catastrophic cascades across interdependent networks [8,9]. However, there is still room for increasing the realism of such models [10], especially in the case of electric grids or gas pipelines. In this paper we move a preliminary step in such direction, trying to capture the systemic effect for coupled networks with long range interactions.

To highlight the possibility of emergent behavior, we will first abstract *PNI*s in order to understand the basic mechanisms that could drive systemic failures; in particular, we will consider finite capacity networks where a commodity (a scalar quantity) is produced at source nodes, consumed at load nodes and distributed as a Kirchhoff flow (e.g. fluxes are conserved). For such systems, we will first introduce a simplified model that is amenable of a self-consistent analytical solution. Subsequently, we will extend such model to the case of several coupled networks and study the cascading behavior under increasing stress (i.e. increasing flow magnitudes).

In Section 2, we develop our simplified model of overload cascades first in isolated (Section 2.2) and coupled systems (Section 2.3). In particular, in Section 2.1, we introduce the concept of flow network with a finite capacity and relate conservation laws to Kirchhoff's equations and to the presence of long range correlation. To account for such correlations, in Section 2.2 we introduce a mean field model for the cascade failures of flow networks; in Section 2.3, we extend the model to the case of several interacting systems. Finally, in Section 3 we discuss and summarize our results.

2. Model

2.1. Flow networks

Let us consider a network $G=(\mathcal{V},\mathcal{E},\mathbf{c})$ where $\mathcal{V}=\{1\le i\le|\mathcal{V}|\}$ is the node set, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges and $\mathbf{c}=\{c_{(i,j)}\}$ is the vector characterizing the capacities of the edges (i,j). We associate to the nodes a vector $\mathbf{p}=\{p_i\}$ that characterize the production ($p_i>0$) or the consumption ($p_i<0$) of a commodity. We further assume that there are no losses in the network (i.e. $\sum_i p_i=0$); hence, the total load on the network is

$$L=\sum_{i:p_i>0} p_i.$$

Algorithm 1 Network cascading

Set initial failures $\mathcal{F}^0$
$t \leftarrow 0$
repeat
  $t \leftarrow t+1$
  Calculate flows $\mathbf{f}^t \leftarrow F(\mathbf{p},G|\mathcal{F}^{t-1})$
  Calculate new failures $\Delta\mathcal{F}^t \leftarrow \{(ij):|f^t_{ij}|>c_{ij}\}$
  $\mathcal{F}^t \leftarrow \mathcal{F}^{t-1}\cup\Delta\mathcal{F}^t$
until $\Delta\mathcal{F}^t\equiv\emptyset$

The distribution of the commodity is described by the fluxes $\mathbf{f}=\{f_{(i,j)}\}$ on the edges $(i,j)\in\mathcal{E}$ that are supposed to respect Kirchhoff equations, i.e.

$$\sum_j f_{(i,j)}=p_i. \tag{1}$$

The relation among fluxes and demand/load is described by constitutive equations

$$\mathbf{f}=F(\mathbf{p},G) \tag{2}$$

where in general Eq. (2) is non-linear but satisfies Eq. (1).

The finite capacity $c_{(i,j)}$ constrains the maximum flux on link (i,j)

$$|f_{(i,j)}|<c_{(i,j)} \tag{3}$$

above which the link will cease functioning. As an example, power lines are tripped (disconnected) when power flow goes beyond a certain threshold. Since flows will redistribute after a link failure, it could happen that other lines get above their flow threshold and hence consequently fail, eventually leading to a cascade of failures. A typical algorithm to calculate the consequences of an initial set of line failures $\mathcal{F}^0=\{(ij)\text{ failed}\}$ is Algorithm 1.

Here $F(\mathbf{p},G|\mathcal{F})$ calculates the flows subject to the constraints that flows are zero in the failure set of edges $(i,j)\in\mathcal{F}$.

To develop a general model that helps us understanding the class of failures that can affect Kirchhoff-like flow networks, let us start from rewriting Eq. (1) in matrix form

$$\mathcal{B}^{\mathcal{T}}\mathbf{f}=\mathbf{p} \tag{4}$$

using the incidence matrix B that associates to each link (i,j) its nodes i and j and vice-versa. B is an $|\mathcal{V}|\times|\mathcal{E}|$ matrix where each column corresponds to an edge (i,j); its columns are zero-sum and the only two non-zero elements have modulus 1 and are on the ith and on the jth row.

The matrix B is related to the Laplacian B^TB of the system; in particular, it shares the same right eigenvalues and the same spectrum (up to squaring operations); hence, it is a long-range operator since perturbation on a node of the system can be reflected on nodes far away on the network [5,6].

2.2. Mean field model for cascades on a single network

Due to the long range nature of Kirchhoff's equations, to understand the qualitative behavior of such networks we can resort to a mean field model of flow networks where one assumes that when a link fails, its flow is re-distributed equally among all other links.