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Border collisions inside the stability domain of a fixed point

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HIGHLIGHTS

- Unraveling the bifurcation structure of a single-phase H-bridge inverter.
- Demonstration of regular structures formed by persistence border-collision curves.
- Detection of qualitatively different regions inside the fixed point stability domain.
- Studies of the processes associated with a new route to chaos in switching systems.

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ABSTRACT

Recent studies on a power electronic DC/AC converter (inverter) have demonstrated that such systems may undergo a transition from regular dynamics (associated with a globally attracting fixed point of a suitable stroboscopic map) to chaos through an irregular sequence of border-collision events. Chaotic dynamics of an inverter is not suitable for practical purposes. However, the parameter domain in which the stroboscopic map has a globally attracting fixed point has generally been considered to be uniform and suitable for practical use. In the present paper we show that this domain actually has a complicated interior structure formed by boundaries defined by persistence border collisions. We describe a simple approach that is based on symbolic dynamics and makes it possible to detect such boundaries numerically. Using this approach we describe several regions in the parameter space leading to qualitatively different output signals of the inverter although all associated with globally attracting fixed points of the corresponding stroboscopic map.

1. Introduction

Power electronic inverter systems (DC/AC converters) provide AC voltage or current of specified amplitude and frequency from a DC source. By virtue of their high efficiency and relatively low costs, inverters have achieved widespread application in modern power engineering. Standard examples of devices that include inverters are uninterruptible power supplies (UPS), active filters, flexible AC transmission systems (FACTS), voltage compensators, and so on [1]. Moreover, in the last few years the interest in inverters has been continuously increasing because of their use in solar panel systems and in the power supply systems of electric and hybrid cars [2–4].

The functioning of inverter systems is characterized by a cyclic switching of the circuit topology. This switching process is controlled through pulse-width modulation in accordance with the desired output wave form [1]. Feedback regulation of the pulse-width modulation provides a way to correct deviations from the desired wave form and, by operating at high switching rates, to keep the output ripple at an acceptable level even with relatively small filter components.

Like other power electronic systems with switching control, inverter systems require us to work with piecewise-smooth models for their adequate description [5,6]. Such models are characterized by a division of their phase space into several regions in which the dynamic behavior is governed by different smooth systems. These regions are separated from each other by so-called switching manifolds. Hence, in addition to the bifurcations known for smooth systems, piecewise-smooth systems display a variety of border-collision related phenomena occurring when an invariant set (such as, for example, a cycle) collides with a switching manifold.

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In relation to the present work, it is necessary to clearly distinguish between two different types of border collisions with widely different outcomes. If the collision causes a change in the topological structure of the phase space it is commonly referred to as a border-collision bifurcation. As an example one can consider the transition from a stable fixed point to a stable n-cycle, $n \geqslant 2$, or to a chaotic attractor. However, it is also possible that the invariant set after the border collision is of the same kind as before. For example, if there are stable fixed points before and after the border collision, then neither the stability nor the periodicity changes but only the location of the fixed point with respect to the switching manifold is affected. In such cases we cannot speak about bifurcations in a strong sense, because a bifurcation is commonly defined as a change in the topological structure of the phase space, which does not occur in the considered situation. Therefore, following the classification introduced by Feigin [7,8], we refer to this type of collision as a persistence border collision. An overview of these, and other, border-collision related phenomena may be found in the book by di Bernardo et al. [9].

The normal operational regime for the considered class of converter systems is the regime of stable period-1 dynamics as prescribed by the (sinusoidal) reference signal. However, as parameters are varied, this period-1 mode may become unstable and the system starts to show oscillations in the form of a small-amplitude high-frequency quasiperiodic [10] or chaotic [11,12] ripple modulated by the low-frequency external reference signal. The onset of chaotic oscillations in a H-bridge inverter with a resistive and inductive load was reported in our recent publication [12]. That article discussed the transition from a stable period-1 mode to deterministic chaos through an irregular cascade of border-collision events. More precisely, the transitional states involved sequences of persistence border collisions interrupted by occasional border-collision bifurcations. In addition, we found that similar high-frequency oscillations modulated by the low-frequency reference signal may appear inside the region of stability for a period-1 mode. From a practical point of view, although these oscillations do not influence the period of the overall signal, they may significantly worsen the harmonic distortion of the load current. This leads to the questions: what is the role of the persistence border collisions in the organization of the internal structure of the stability region for period-1 dynamics and how to determine the boundary of the normal operational regime?

In the present paper we consider the same inverter as in [11,12], but the focus of our study is now on that part of the parameter space where the stable period-1 mode is globally attracting. This region of parameter space is generally assumed to be fairly uniform and without significant internal structure. In reality, however, the region contains an interesting and rather complicated structure formed by persistence border-collision curves, a structure that, to the best of our knowledge, has never been reported before. To detect this structure we developed a simple numerical approach that allows us to locate border collisions of any type, including persistence collisions. Since neither the stability nor the periodicity of the operational mode changes in the persistence border collisions, the approach has to be based on symbolic dynamics. Using this approach we outline the interior structure of the domain in the parameter space where the fixed point is globally attracting. In particular, we show how the quality of the output signal of the inverter may change (preserving the required period of the sinusoidal reference signal) from being almost perfect in certain regions to becoming practically unacceptable in others. In this connection it is interesting to note that the structure of the border-collision network depends on whether the inverter operates with odd or even values of the so-called frequency modulation ratio m that measures the number of switching cycles during one period of the reference cycle. Finally, we relate the large scale oscillatory variation in the transition from regular to chaotic dynamics to the characteristic structure of the network of border-collisions curves.

The purpose of the present study is twofold: From the theoretical point of view, we want to report how persistence border collisions may form structures in 2D parameter space. The present paper does not present any general results or rigorous proofs, but only reports a few examples of the observed structures and describes their regularities. Although we have already observed similar structures in other systems, the task to investigate the conditions leading to their appearance is postponed for future work. On the other side, from the practical point of view, we want to show in what part of the parameter region the quality of the output signal is acceptable and in which part it is not. Due to its simplicity, the numerical approach reported in this work can be applied without modifications to a broad class of DC/AC and AC/DC converter systems.

2. Description of the system

2.1. PWM H-bridge single-phase inverter

Fig. 1(a) shows a schematic diagram of the considered pulse-width modulated H-bridge single-phase inverter and Fig. 1(b) illustrates the generation of the switching signal used to control the four switches S_1–S_4.

The four switches of the bridge structure operate in pairs such that S_1 and S_4 are closed when S_2 and S_3 are open, and *vice versa*. When S_1, S_4 are on and S_2, S_3 are off, a positive voltage E_0 is applied to the load, and when S_1, S_4 are off and S_2, S_3 are on, this voltage is reversed. The switches are controlled by the sinusoidal PWM modulator through a feedback mechanism. In order to generate the switching signals to the switches S_1, S_4 and S_2, S_3, the corrector amplifier DA_2 first determines the error signal $\xi(t) = \alpha(V_{\text{ref}}(t) - V_{\text{cs}}(t))$ that measures the difference between the reference sinusoidal voltage $V_{\text{ref}}(t) = V_m \cdot \cos(2\pi t/T)$ and the output voltage $V_{\text{cs}}(t) = \beta i(t)$ of the current sensor CS. Here, α is the corrector gain factor and β is referred to as the current sensor sensitivity; V_m and $T = ma$ are the amplitude and the period of the reference signal, respectively. The parameter a denotes the ramp period (the period of the clock signal V_{clock}) and, as previously introduced, m is referred to as the frequency modulation ratio, i.e., the number of clock cycles during the period T of the reference signal. The frequency modulation ratio m obviously plays an important role in determining the accuracy with which the reference signal can be reproduced by the load current.

As illustrated in Fig. 1(b), the sample-and-hold unit S/H reads the error signal $\xi(t)$ at every clock time $t = ka$, $k = 0, 1, 2 \ldots$, and maintains it for the following switching period. This produces the control signal $V_{\text{con}}(t)$. Finally the comparator DA_1 compares this control signal from the sample-and-hold unit with a periodic ramp function $V_{\text{ramp}}(t)$ in order to generate the switching signals to the switches S_1, S_4, and S_2, S_3. As long as $V_{\text{con}}(t) > V_{\text{ramp}}(t)$, switches S_1, S_4 are on and S_2, S_3 are off, while S_1, S_4 are off and S_2, S_3 on for $V_{\text{con}}(t) \leqslant V_{\text{ramp}}(t)$. This type of modulation is also known as pulse-width modulation of the first kind.

The ramp function $V_{\text{ramp}}(t)$ varies from $-V_0$ to $+V_0$ and in synchrony with the clock signal. If $V_{\text{con}}(t) \geqslant +V_0$ or $V_{\text{con}}(t) \leqslant -V_0$ the modulator is saturated. In the first case, i.e., if $V_{\text{con}}(t) \geqslant +V_0$, the duration of the positive pulse is equal to the ramp period a (see the time intervals $2a < t \leqslant 3a$ and $3a < t \leqslant 4a$ in Fig. 1(b)), and in the second case (i.e., $V_{\text{con}}(t) \leqslant -V_0$) it is equal to zero as can be observed in Fig. 1(b) during the time intervals $7a < t \leqslant 8a$ and $8a < t \leqslant 9a$.