Chaos, Solitons and Fractals 89 (2016) 2–7

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Fuzzy-valued linguistic soft set theory and multi-attribute decision-making application

Zhao Aiwu [a,*], Guan Hongjun [b]

[a] School of Management, Jiangsu University, Zhenjiang, China

[b] School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan, China

ARTICLE INFO

Article history:
Available online 9 October 2015

Keywords:
Soft set
Fuzzy set
Linguistic variables
Multiple attribute decision making
Generalized weighted aggregation
Fuzzy-valued linguistic soft set

ABSTRACT

In this work, we propose the theory of fuzzy linguistic soft set (FLSS) to represent the uncertainty and multi-angle of view when decision makers evaluate an object during decision-making. FLSS integrates fuzzy set theory, linguistic variable and soft set theory. It allows decision makers to utilize linguistic variables to evaluate an object and utilize fuzzy values to describe the corresponding grade of their support of their decisions. Meanwhile, because of the flexibility of soft set, decision makers can use more than one pair of fuzzy-linguistic evaluations to express their opinions from multiple perspectives directly, if necessary. Therefore, it is more flexible and practical than traditional fuzzy set or 2-dimension uncertainty linguistic variable. We also develop a generalized weighted aggregation operator for FLSSs to solve corresponding decision-making issues. Finally, we give a numerical example to verify the practicality and effectiveness of the proposed method.

1. Introduction

Since Zadeh proposed his remarkable theory of fuzzy sets (FSs) in 1965 [1], FSs have been widely used in various fields to deal with uncertainty problems during decision making [2–4]. However, in some cases, using one single specific value to represent the grade of membership of the fuzzy set is difficult. Therefore, Atanassov extended Zadeh's traditional FSs, and put forward the theory of intuitionistic fuzzy sets (IFSs) [5]. IFSs took into account the membership degree, nonmembership degree, and the degree of hesitation simultaneously. It is more flexible and practical in dealing with fuzziness and uncertainty than traditional FSs. Later on, IFSs were extended to interval valued intuitionistic fuzzy sets (IVIFSs) [6], hesitant fuzzy sets (HFSs) [7], etc.

However, in many decision making problems, real numbers are difficult to measure the attribute values of the alternatives, and decision makers prefer to use linguistic evaluation for an object, such as very good, good, normal, bad, very bad, and so on. Hence, Zedah (1975) [8] introduced the definition of linguistic variable and applied it to the fuzzy reasoning. In order to accurately reflect the reliability of the linguistic variables, Zhu et al. [9] proposed the 2-dimension linguistic evaluation method to deal with linguistic fuzzy decision problems, such as blind review of journal papers, etc. Using 2-dimension linguistic evaluation method, decision makers can describe their evaluation in a 2-dimensional linguistic variable. One is used to describe linguistic evaluation result for an attribute, and the other is used to describe the subjective evaluations of the reliability of their given results. Later on, research on multi-attribute group decision making based on 2-dimension linguistic information is developed [10–12]. 2-dimension linguistic variable can help people to express their opinions for attributes which cannot be measured by real numbers.

* Corresponding author. Tel.: (86)+0511-88780186.
E-mail addresses: aiwuzh@126.com (Z. Aiwu), jjxyghj@126.com (G. Hongjun).

http://dx.doi.org/10.1016/j.chaos.2015.09.001

In fact, when decision makers evaluate the attribute of one of the alternatives, he may provide different evaluation results from different perspectives of the attribute of the alternative. For example, when evaluating the location attribute for one of the alternatives, from the perspective of traffic convenience, the answer is good; while from the perspective of the number of surrounding residents, the answer is bad. In this case, obviously, one linguistic variable is not enough. Hence, we propose fuzzy-valued linguistic soft sets (FLSSs) to overcome such kinds of difficulties. Soft set theory was proposed by Molodtsov [13] in 1999. Using soft set theory, people can choose parameters and their forms according to their needs. It can be more flexible to describe the uncertainty and imperfection of the objective world. Researchers have done many interesting works by combining soft set theory with the ideas of fuzzy sets, rough sets or intuitionistic fuzzy sets [14–17], where soft sets are used to represent the characteristics of the described object.

In this paper, we proposed FLSS theory based on both soft sets and fuzzy sets, and define the assessment linguistic variables as the parameters of the soft sets. Decision makers can choose one or more of the linguistic variables to evaluate the attribute of the alternative from different perspectives, and decide the membership degree of each linguistic evaluation respectively. Therefore, FLSS extends the function of 2-dimension linguistic method both in flexibility and accuracy. It can more accurately reflect the evaluation of decision makers on objects.

The main contributes of our proposal are listed below:

(1) We define a FLSS form of evaluation which integrates linguistic variable as evaluation, fuzzy number as membership degree of corresponding linguistic variable, and soft set structure to let decision makers to evaluate an object from different perspectives directly if necessary. So, it is more flexible, and can be regarded as a general evaluation model.
(2) In traditional multi-attribute decision making problems, decision makers must judge the degree of every attribute for each alternative. Employing FLSS form, it is no need for decision makers to answer all of the questions. We can collect assessments from different decision makers using FLSS form. So, it is not only effective for different perspective evaluation, but also effective for group decision making with incomplete information.
(3) We develop a FLSSGWA operator to aggregate different FLSSs, and validate its effectiveness. FLSSGWA operator maintains most of the information of each FLSS.

The reminder of this paper is organized as follows. In Section 2, we introduce the concept of FLSS, define the basic operational rules, and develop a generalized weighted aggregation operator based on traditional generalized weighted aggregation method for real numbers and the operational rules of FLSS. In Section 3, we apply the defined operators to solve multiple attribute decision making problems. In Section 4, we illustrate the decision making steps based on the proposed method. Section 5 is the conclusion of this paper.

2. Fuzzy-valued linguistic soft set

2.1. The definition of fuzzy-valued linguistic soft set

Suppose that $S=\{s_0, s_1, \ldots, s_{l-1}\}$is a finite and fully ordered discrete term set, where l is an odd number. In real situations, l would be equal to 3, 5, 7, 9, etc. For example, when l=7, it is represented as follows:

$$\begin{aligned} S &= \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} \\ &= \{\text{very bad}, \text{bad}, \text{below fair}, \text{fair}, \text{above fair}, \text{good}, \text{very good}\}. \end{aligned}$$

For any linguistic set $S=\{s_0, s_1, \ldots, s_{l-1}\}$, the relationship between the element s_i and its subscript i is strictly monotonically increasing [18], so the function can be defined as follows: $f: s_i = f(i)$. Clearly, the function $f(i)$ is a strictly monotonically increasing function about a subscript i. To preserve all of the given information, the discrete linguistic label $S=\{s_0, s_1, \ldots, s_{l-1}\}$ is extended to a continuous linguistic label $\bar{S}=\{s_\alpha | \alpha\in R\}$, which satisfies the above characteristics.

Definition 1. Let U be an initial universe set and $S=\{s_0, s_1, \ldots, s_{l-1}\}$ be a linguistic assessment set. Let $I=[0,1]$, and I^U denote the set of all fuzzy subsets of U. Consider a nonempty set A, $A\subseteq S$. A pair (K, A) is called a fuzzy-valued linguistic soft set (for short FLSS) over U, where K is a mapping given by K: $A\rightarrow I^U$. It is also represented as $\forall s_a \in A$.

$$K(s_a) = \left\{\left\langle x, \mu_{K(s_a)}(x)\right\rangle | x\in U\right\} \in I^U \tag{1}$$

where $\mu_{K(s_a)}(x) \in [0, 1]$, it represents the membership degree of element $x(\in U)$belonging to linguistic assessment $s_a(\in A)$.

2.2. The operational rules of FLSS

Definition 2. Suppose (F, A) and (G, B) are two FLSSs over U, their intersection and union operation are defined as follows.

(1) $(F, A)\cap(G, B) = (H, C)$, where $C=A\cap B$, and for $\forall s_c\in C$,

$$H(s_c) = F(s_c)\cap G(s_c) \tag{2}$$

(2) $(F, A)\cup(G, B) = (M, C)$, where $C=A\cup B$, and for $\forall s_c\in C$,

$$M(s_c) = \begin{cases} F(s_c), & s_c\in A-B \\ F(s_c)\cup G(s_c), & s_c\in A\cap B \\ G(s_c), & s_c\in B-A \end{cases} \tag{3}$$

Definition 3. Suppose (F, A) is a FLSS over U, $(F, A)^C = (F, A^C)$ is defined as the complement of (F, A), where for $\forall s_a\in A$, ${s_a}^c = s_{l-1-a}$. l is the number of linguistic variables as defined by Definition 1. And for $\forall {s_a}^C \in A^C$,

$$\mu_{F({s_a}^C)} = \mu_{F(s_a)} \tag{4}$$

Definition 4. Suppose (F, A), (G, B) are two FLSSs over U, when $\lambda > 0$, their operational rules are defined as follows.

$$(1)\quad (F, A)\oplus(G, B) = (F, A)\cup(G, B) \tag{5}$$

$$(2)\quad (F, A)\otimes(G, B) = (F, A)\cap(G, B) \tag{6}$$