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Chua's circuit model with Atangana–Baleanu derivative with fractional order

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ABSTRACT

The analysis of circuit employing the Kirchhoff's circuit laws also known as dynamics of Chua's circuit is extended in this work using the newly established fractional derivative with nonlocal and non-singular kernel. A new numerical analysis is presented and used to solve the extended model. Some numerical simulations are done for different values of the fractional order and new chaotic behaviors are obtained.

1. Introduction

A three nonlinear ordinary differential equation in the variables $x(t)$, $y(t)$, $z(t)$ standing for the voltages transversely the capacitors $C1$ and $C2$ are used to describe the circuit using Kirchhoff's circuit laws [1,2]. This dynamic circuit is also known as Chua's circuit. It was suggested in [3] that any self-governing electrical path obtained via standard components namely inductors; resistors and capacitors have to meet three characteristics in order to portray chaotic behavior. The three characteristics suggested in [3] are

1. One or more nonlinear elements.
2. One or more locally active resistors.
3. Three or more energy-storage elements.

It is well known that Chua's circuit is one of the easiest electronic paths satisfying the above criteria [3]. Many researchers have done some works using this model; particularly some authors have extended this model using the Caputo derivative with fractional order [4–6]. Although a clear reason for why the fractional derivative was used to extend the model was not presented in these papers, but new chaotic behavior were obtained from their numerical simulations. Recently a work done by Machado and Ortiguiera suggested that, an operator will be called fractional derivative if in addition of the criteria prescribed in their paper [7], the operator is able to portray or exhibit some chaotic behavior. Recently Atangana and Baleanu, in order to solve the problem of fractional derivative with non-singular and nonlocal kernel suggested a new derivative based on the generalized Mittag–Leffler function [8,9]. Atangana and Koca used this new derivative and extended the Lorenz Attractor model and obtained very good results [9]. In this paper the Atangana–Baleanu derivative with fractional order will be used to further examine chaotic behavior on the Chua's circuit model. The classical Chua's circuit model consider here is given as

$$\begin{cases} \frac{dx(t)}{dt} = \alpha[y(t) - x(t) - f(x(t))], \\ RC_2\frac{dy(t)}{dt} = x(t) - y(t) + Rz(t), \\ \frac{dz(t)}{dt} = -\beta y(t), \\ f(x(t)) = mx(t) + \frac{1}{2}(n-m)(|x(t)+1| - |x(t)-1|). \end{cases} \tag{1}$$

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The above model was obtained in [10]. The function $f(x)$ demonstrates the electrical response of the nonlinear resistor [10] also this function shape depends on the fastidious pattern on its components. α and β are parameters determined by the particular values of the circuit components [10]. The Chua's model is a useful system to investigate more original and applied chaos theory. Due to the usefulness of this model, it has been a subject of many investigations in the literature. Nonetheless, no study has been done using the newly established fractional derivative.

2. Atangana–Baleanu derivative with fractional order

In this section, the Atangana–Baleanu derivative with fractional order and their properties are presented to accommodate those readers that are not aware of it.

Definition 1. Let $f \in H^1(x, y)$, $y > x$, $\alpha \in [0, 1]$ and not necessary differentiable then, the definition of the new fractional derivative (Atangana–Baleanu fractional derivative in Riemann–Liouville sense) is given as [8,9]

$$ {}_{l}^{ABR}D_t^{\alpha}[g(t)] = \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}\int_l^t g(k)E_{\alpha}\left[-\frac{\alpha}{1-\alpha}(t-k)^{\alpha}\right]dk, \tag{2}$$

Noting that

$$E_{\alpha}(t^{\alpha}) = \sum_{\upsilon=0}^{\infty}\frac{t^{\alpha\upsilon}}{\Gamma(\alpha\upsilon+1)} \tag{3}$$

Definition 2 [8,9]**.** $f \in H^1(x, y)$, $y > x$, $\alpha \in [0, 1]$ with the function f differentiable then, the definition of the new fractional derivative (Atangana–Baleanu derivative in Caputo sense) is given as

$$ {}_{l}^{ABC}D_t^{\alpha}[g(t)] = \frac{B(\alpha)}{1-\alpha}\int_l^t \frac{d}{dk}[g(k)]E_{\alpha}\left[-\frac{\alpha}{1-\alpha}(t-k)^{\alpha}\right]dk, \tag{4}$$

here, the function B satisfies the following $B(0)=B(1)=1$ [8,9].

Definition 3 [8,9]**.** The fractional integral associate to the new fractional derivative with nonlocal kernel (Atangana–Baleanu fractional integral) is defined as

$$ {}_{b}^{AB}I_t^{\alpha}[d(t)] = \frac{1-\alpha}{B(\alpha)}d(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_b^t d(j)(t-j)^{\alpha-1}dj \tag{5}$$

When alpha is zero we recover the initial function and if also alpha is 1, we obtain the ordinary integral.

3. Modified Chua's circuit

The concept of fractional derivative has been employed to obtain many chaotic behavior in many research papers in the literature. It has been demonstrated that several chaotic systems will stay chaotic when their models are extended within the scope of fractional order derivatives [10]. It was demonstrate that the Chua's model with fractional derivative with a lower order 2.7 was able to replicate a chaotic attractor. The model of Wien bridge oscillator was investigated within the scope of fractional calculus, and the results obtained therein showed that a limit cycle could be replicated for any fractional order, with an adequate value of the amplifier gain [11]. The jerk model was studied using the concept of fractional derivative and chaotic attractor was obtained with the system order as low as 2.1. In this paper in order to check the effect of Mittag–Leffler nonlocal and non-singular kernel into the definition of fractional derivative, the time derivative of Eq. (1) will be replaced by the time Atangana–Baleanu fractional derivative to obtain

$$\begin{cases} {}_{0}^{ABC}D_t^{\alpha}[x(t)] = \alpha[y(t) - x(t) - f(x(t))], \\ RC_2{}_{0}^{ABC}D_t^{\alpha}[y(t)] = x(t) - y(t) + Rz(t), \\ {}_{0}^{ABC}D_t^{\alpha}[z(t)] = -\beta y(t), \\ f(x(t)) = mx(t) + \frac{1}{2}(n-m)(|x(t)+1| - |x(t)-1|). \end{cases} \tag{6}$$

For simplicity, we let

$$\begin{cases} {}_{0}^{ABC}D_t^{\alpha}[x(t)] = F_1(x, y, z, t), \\ RC_2{}_{0}^{ABC}D_t^{\alpha}[y(t)] = F_2(x, y, z, t), \\ {}_{0}^{ABC}D_t^{\alpha}[z(t)] = F_3(x, y, z, t). \end{cases} \tag{7}$$

Applying the Atangana–Baleanu fractional integral on both sides of system (7) we obtain

$$\begin{cases} x(t) = x_0(t) + \frac{1-\alpha}{B(\alpha)}F_1(x, y, z, t) \\ \qquad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t F_1(x, y, z, \tau)(t-\tau)^{\alpha-1}d\tau, \\ y(t) = y_0(t) + \frac{1}{RC_2}\frac{1-\alpha}{B(\alpha)}F_2(x, y, z, t) \\ \qquad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t F_2(x, y, z, \tau)(t-\tau)^{\alpha-1}d\tau, \\ z(t) = z_0(t) + \frac{1-\alpha}{B(\alpha)}F_3(x, y, z, t) \\ \qquad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t F_3(x, y, z, \tau)(t-\tau)^{\alpha-1}d\tau. \end{cases} \tag{8}$$

One of the aim of our investigation is to solve the above equation numerically, therefore to achieve this, we first present the numerical approximation of the Atangana–Baleanu fractional integral. The technique used here is the predictor-corrector scheme. By definition we have that

$$ {}_{b}^{AB}I_t^{\alpha}[d(t)] = \frac{1-\alpha}{B(\alpha)}d(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_b^t d(j)(t-j)^{\alpha-1}dj$$

Then let us consider $h = \frac{T}{N}$, $t_l = hl(l = 0, 1, 2, 3.....N)$ where T is considered to be the upper bound of the interval within which the solution is being investigated. The corrector formula for Atangana–Baleanu fractional integral or the Volterra version of a given partial differential equation is given as

$$d_h(t_{n+1}) = d_0(t_{n+1}) + \frac{1-\alpha}{B(\alpha)}F(t_{n+1}, d_h^p(t_{n+1})) + \frac{\alpha}{B(\alpha)}\begin{Bmatrix} \frac{h^{\alpha}}{\Gamma(\alpha+2)}F(t_{n+1}, d_h^p(t_{n+1})) + \\ \frac{h^{\alpha}}{\Gamma(\alpha+2)}\sum_{i=0}^{n}\delta_{i,n+1}F(t_i, d_h^p(t_j)) \end{Bmatrix} \tag{9}$$