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H. Darabi, F. Saeedi, M. Eshrati

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A CHARACTERIZATION OF FINITE DIMENSIONAL NILPOTENT FILIPPOV ALGEBRAS

H. DARABI, F. SAEEDI, AND M. ESHRATI

ABSTRACT. Let A be a nilpotent Filippov (n-Lie) algebra of dimension d and put $s(A) = \binom{d-1}{n} + n - 1 - \dim \mathcal{M}(A)$ and $t(A) = \binom{d}{n} - \dim \mathcal{M}(A)$, where $\mathcal{M}(A)$ denotes the multiplier of A. The aim of this paper is to classify all nilpotent n-Lie algebras A for which $s(A) = 0$, 1 or 2, and applying it in order to determine all nilpotent n-Lie algebras A satisfying $0 \leq t(A) \leq 8$.

1. INTRODUCTION

In 1985, Filippov [12] introduced the concept of *n-Lie (Filippov) algebras*, as an n-ary multilinear and skew-symmetric operation $[x_1, \ldots, x_n]$, which satisfies the following generalized Jacobi identity

$$[[x_1, \ldots, x_n], y_2, \ldots, y_n] = \sum_{i=1}^{n} [x_1, \ldots, [x_i, y_2, \ldots, y_n], \ldots, x_n]].$$

Clearly, such an algebra becomes an ordinary Lie algebra when $n = 2$. Beside presenting many examples of n-Lie algebras, he also extended the notions of simplicity and nilpotency and determined all $(n+1)$-dimensional n-Lie algebras over an algebraically closed field of characteristic zero. There are a great deal of difference between n-Lie algebras and the ordinary Lie algebras. For example, the famous Jacobson's theorem and the root theory of describing semi-simple Lie algebras are not valid any more when studying n-Lie algebras. The study of n-Lie algebras is important since it is related to geometry and physics. Among other results, n-Lie algebras are classified in some cases. For example, Bai et. al in [3] classify all n-Lie algebras of dimension $n+1$ over a field of characteristic 2. Also, they show that there is no simple n-Lie algebra of dimension $n+2$. (see [1], [6], [11], [13], [18] and [19] for more information on the Filippov algebras).

In 1986, Kasymov [16] introduced the notion of nilpotency of an n-Lie algebra as follows:

An n-Lie algebra A is *nilpotent* if $A^s = 0$ for some non-negative integer s, where A^i is defined inductively by $A^1 = A$ and $A^{i+1} = [A^i, A, \ldots, A]$. The ideal $A^2 = [A, \ldots, A]$ is called the *derived subalgebra* of A. The *center* of A is defined by

$$Z(A) = \{x \in A : [x, A, \ldots, A] = 0\}.$$

Let $Z_0(A) = \langle 0 \rangle$. Then the ith center of A is defined inductively by

$$\frac{Z_i(A)}{Z_{i-1}(A)} = Z\left(\frac{A}{Z_{i-1}(A)}\right)$$

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