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Dynamics of universal computation and $1/f$ noise in elementary cellular automata

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ABSTRACT

Elementary (one-dimensional two-state three-neighbor) cellular automaton rule 110 is capable of supporting universal computation and exhibits $1/f$ noise. However, the power spectra of rule 110 do not always exhibit $1/f$-type spectrum because of periodic background particular to rule 110. Since periodic background in rule 110 does not play any role in supporting universal computation, we can expect that the removal of periodic background does not change the behavior essential to supporting universal computation. As a result of removing periodic background, we found that the power spectra fit better to $1/f$ noise. This result suggests $1/f$ noise is an intrinsic property of computational universality in cellular automata.

1. Introduction

Cellular automaton (CA) is a d-dimensional array with a finite automaton residing at each site. Each automaton called cell takes the states of neighboring cells as input and makes the transition of its state according to a set of transition rules. CAs are also considered to be spatially and temporally discrete dynamical systems with large degrees of freedom. It was proved that elementary CA (ECA), namely one-dimensional and two-state, three-neighbor CA rule 110 is computationally universal [1]. Computational universality is an ability to perform any algorithms like Turing machines or conventional digital computers can do so. In addition, the evolution of rule 110 starting from a random initial configuration exhibits $1/f$ noise [2]. The fluctuation whose power spectrum $S(f)$ is inversely proportional to frequency f is called $1/f$ noise [3]. In spite of its ubiquity, its origin is not well understood yet. Another example of computationally universal CA with $1/f$ noise is the Game of Life (LIFE). LIFE is a two-dimensional and two-state, nine-neighbor outer totalistic CA. LIFE is capable of supporting universal computation [4,5], while the evolution starting from a random initial configuration has $1/f$-type spectrum [6]. These results suggest that there is a relationship between computational universality and $1/f$ noise in CAs.

However the range of frequencies in the power spectra of ECA rule 110 fitting to power law is not broad. For example, the range of frequencies fitting to power law is $f = 1 - 10$ in the evolution of 700 cells for 1024 time steps [2]. It seems that this is caused by the periodic background typically observed in the evolution of rule 110. Since the periodic background does not play an essential role for performing computation, we can guess that the essential feature in the evolution of rule 110 as a computing process is not impaired by the removal of the periodic background and that $1/f$-type spectrum becomes clear by removing the periodic background.

In this paper we study the influence of the removal of periodic background on the shape of power spectrum of several ECA rules. In Section 2 we explain the method of spectral analysis of ECA. In the following three sections we deals with ECA rule 110 as well as rule 54 and rule 62 which exhibit power law in power spectrum. In the final

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section we discuss the relationship between computational universality and $1/f$ noise in CAs.

2. Spectral analysis of cellular automata

Let $s_x(t) \in \{0, 1\}$ be the value of site x at time step t in an ECA. The site value evolves by iteration of the mapping,

$$s_x(t+1) = F(s_{x-1}(t), s_x(t), s_{x+1}(t)). \tag{1}$$

Here F is an arbitrary function specifying the set of ECA rules. The set of ECA rules is determined by a binary sequence with length $2^3 = 8$,

$$F(1,1,1)F(1,1,0)\cdots F(0,0,0). \tag{2}$$

Therefore the total number of possible distinct sets of ECA rules is $2^8 = 256$ and each set of rules is abbreviated by the decimal representation of the binary sequence (2). Out of the 256 ECA rules 88 of them remain independent (appendix of [7]).

The Discrete Fourier Transform of a time series of states $s_x(t)$ of site x for $t = 0, 1, \ldots, T-1$ is given by

$$\hat{s}_x(f) = \frac{1}{T}\sum_{t=0}^{T-1} s_x(t) \exp\left(-i\frac{2\pi t f}{T}\right) \quad f = 0, 1, \ldots, T-1. \tag{3}$$

The power over all sites is defined as

$$S(f) = \sum_{x=0}^{N-1} |\hat{s}_x(f)|^2, \tag{4}$$

where N denotes the total number of sites and the summation is taken in all sites. The period of the component at a frequency f in a power spectrum is given by T/f. Throughout this paper we employ periodic boundary conditions where the sites are connected in a circle and each array is started from a random initial configuration where each site takes state zero or state one randomly with independent equal probability.

The spectral analysis on the evolution of 88 independent ECAs starting from random initial configuration revealed that the power spectra of most of the rules are either white noise or Lorenzian type and that rule 54, rule 62 and rule 110 has the power spectra of power law and especially rule 110 exhibits $1/f$ noise during the longest time steps [2]. Therefore we focus on the power spectra of these three rules in subsequent sections.

3. Rule 110

The rule function of rule 110 is given by the following:

$$\frac{111}{0}\ \frac{110}{1}\ \frac{101}{1}\ \frac{100}{0}\ \frac{011}{1}\ \frac{010}{1}\ \frac{001}{1}\ \frac{000}{0}.$$

The upper line represents the 8 possible states of neighborhood and the lower line specifies the state of the center cell at the next time step. Cook proved the computational universality of rule 110 by showing that rule 110 can emulate cyclic tag system (CTS) [1]. A CTS is an abstract machine that has an infinitely long tape divided into cells. Each cell can contain a symbol,'0' or '1'. At each step, the CTS removes one symbol from the front of the tape and if that is '1', then it appends the appendant to the end of the tape according to a fixed appendant table that defines the machine, while an '0' causes the appendant to be skipped. An appendant is cyclically chosen from the appendant table. The system halts if the string on the tape is empty. CTSs are proved to be computationally universal.

For example, the transition of the initial word '1' with the appendant table (1, 101) is given as following:

$$\begin{array}{lllllll} & 1 & & & & & \\ \vdash & 1 & & & & & \\ \vdash & & 1 & 0 & 1 & & \\ \vdash & & & 0 & 1 & 1 & \\ \vdash & & & & 1 & 1 & \\ \vdash & & & & & 1 & 1 \\ \vdash & & & \cdots & & & \end{array} \tag{5}$$

Fig. 1 shows the part of the space–time pattern of computing process of a CTS implemented on the array of rule 110. Space–time pattern shows configurations obtained at successive time steps on successive horizontal lines from the top to the bottom in which black squares represent state one sites, white squares state zero sites. A tape data '1' is under construction by the collisions between appendant table data coming from the right and propagating patterns called 'ossifier' coming from the left in Fig. 1. A detailed explanation of the emulation of CTS by rule 110 is found in [8]. The initial configuration file for the CTS emulation by rule 110 can be downloaded at the web site [9].

The left side of Fig. 2 shows a typical example of the space–time pattern of rule 110 starting from a random initial configuration of 200 cells for 200 time steps. We can observe periodic background of small white triangles with period seven in the space–time pattern.

The left side of Fig. 3 is the power spectrum of rule 110 calculated from the evolution starting from a random initial configuration of 4000 cells for 4096 time steps. There are peaks at $f = 585$ (period:7) and its harmonics in the spectrum. The exponent β of power spectrum is estimated

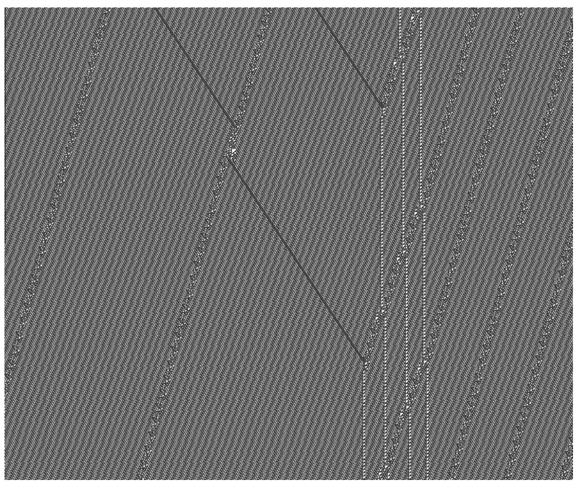

Fig. 1. Space–time pattern of computing process of a CTS implemented on the array of rule 110.