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Quasi-synchronization dynamics of coupled and driven plasma oscillators

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ABSTRACT

The dynamics of two coupled and periodically driven plasma oscillators is investigated in this paper. It is shown that the two oscillators exhibit rich dynamical transition to quasi-synchronized state. Stability and sufficient criteria for synchronization are analytically obtained using linear matrix inequality (LMI) and the Routh–Hurwitz criterion; and qualitatively characterized by the system's interaction energies. Moreover, the transition dynamics is rich with abundant complex bifurcation structures, including Hopf bifurcations. Based on the method of multiple time-scale, steady state equations for the vibration of the coupled oscillators have also been obtained, and bi-resonance induced by coupling at distinct frequencies are reported.

1. Introduction

Coupled nonlinear oscillators are ubiquitous in nature and well known for their rich varieties of dynamical behavior which includes higher-order chaos (hyperchaos), multi-stability of attractors and synchronization. While in general, synchronization phenomena are of fundamental importance in the study of biological, physical and technological problems [1–3]; multi-stability of attractors has been shown to accompany some synchronization transitions [4–6]. The history of synchronization dates back to the earlier observation of synchronization in the popular two clock pendula by Huygens [7]. A similar observation in the output of adjacent organ pipes was reported by Bleckman, whereby the individual effects of the pipes reduce collective output to either silence or peak [1,8]. These earlier studies were more specific to regular or periodic oscillations. In 1990, Pecora and Carroll [9] presented their results on the synchronization of identical chaotic systems which opened new directions of research activities in chaos synchronization [2]. The study of synchronized dynamics of chaotic systems derived its motivations from several potential applications in secure communication systems, time series analysis, modeling brain and cardiac rhythm activity and earthquake dynamics [1,2,10].

In recent years, full (identical/complete) synchronization have been extensively studied in the context of many specific problems. For general coupled chaotic systems with two phase space trajectories $x(t)$ and $y(t)$, the fulfillment of the condition, $\lim_{t\to\infty}\|x(t)-y(t)\|=0$ signifies full synchronization between $x(t)$ and $y(t)$. In reality and for many practical cases, the limit does not always approach zero asymptotically but a constant value, ϵ according to the inequality, $\lim_{t\to\infty}\|x(t)-y(t)\|<\epsilon$, implying imperfect complete or practical synchronization [8,11,12]; which in most cases arises from parameter mismatches between the two coupled systems. In practical situations, parameter mismatch is inevitable in synchronization implementations [13–15], and it has significant effects on the collective

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behaviors of coupled systems. Loss of synchronization may take place in some cases [14]; while in other cases, complete synchronization can be maintained even with large parameter mismatch [15]. Imperfect complete synchronization phenomena are intriguing in nature as well as in practical situations and therefore deserves to be understood. Despite the huge results so far on other forms of synchronization, only few works have reported on quasi-synchronization due partly to its rare occurrence (See for instance Ref. [13,16–19]). More recently, renewed interest has been devoted to investigate this phenomenon and has been reported in [20–22]. In this paper, we report our observations of quasi-synchronization behavior in a model of coupled plasmas exhibiting chaotic dynamics.

Chaotic behavior in nonlinear plasma oscillations has been observed in several theoretical and experimental investigations [23–28]. For instance, the early work of Keen and Fletcher [23] showed that a marginal ion sound instability in a plasma behaves in a chaotic manner similar to that predicted by a Van der Pol type of nonlinear system. Hur et al. [24] observed period-doubling route to chaos in an ion beam going through dusts and electrons without collision; and bounded by two electrodes. Intermittent chaotic behavior in nonlinear three-wave model of space plasma was reported by Miranda et al. [25]. Sheridan [26] reported experimental observation of chaotic dynamics in a complex (dusty) plasma of three particles [27]. Recently, Enjieu-Kadji et al. [29,30] showed that the plasma oscillation maybe modeled as a nonlinear anharmonic oscillator of the Duffing-type displaying varieties of chaotic domain; and more recently considered chaos control problem for the same system [31]. Earlier investigations on the problem of chaos suppression in plasmas systems were experimentally carried out by Bezruchko et al. [32] and Koronovski et al. [33] in which by tuning the frequency of the external signal, chaos suppression was realized. In another related investigation, Viana et al. [28], analyzed the fractal structures of nonlinear plasma, emphasizing that fractals appear in certain applications in plasma physics, like the magnetic field line behavior in tokamaks with ergodic limiters. Viana et al. [28] further demonstrated the observable consequences of fractal structures in terms of the transport properties in the plasma edge of tokamaks and also discussed the role of the fractal structures in the understanding of mesoscale phenomena in plasmas.

Notwithstanding the increasing interest in plasma chaos research, only few studies have been devoted to the investigation of chaos synchronization in a nonlinear plasmas [34–37]. For instance Filatov et al. [35] investigated chaotic synchronization regimes for coupled spatially extended beamplasma Pierce systems and observed different synchronization regimes, namely phase synchronization, generalized synchronization, time-scale synchronization and complete synchronization. Similarly, an impulsive synchronization was employed by Li et al. [36] to realize chaos synchronization in a laser plasma system.

In this paper, we investigate the synchronization behavior of two linearly coupled plasma oscillators with a single external periodic driving. The single periodic forcing could be realized using some physical mechanism such as an externally applied electric field or an ultraviolet light. For instance, by means of photodetachment, ultraviolet light which can extract electrons from materials can be used as an external force to control the charge transport on a dust particle. This is because, it is well known that the transport of dust particles into plasma is proportional to the dust charge and the coagulation of small particles into larger ones due to the attraction or repulsion between charged particles through the coulomb potential (See Ref. [29] and references therein). In general, nonlinear systems with single external periodic driving exhibit varieties of nontrivial complex dynamics [38,39], including the possibility of mode locking phenomenon occurring at rational multipliers of driving frequency [2], among others. These are relevant nonlinear characteristics which are of immense practical applications, in areas such as optical tomography as well as communications [40]. Here, we report the occurrence of quasi-synchronization phenomenon in two coupled and driven plasma oscillators, in which the defining limit of synchronization is bounded within a definite small region around zero. i.e. $\lim_{t\to\infty}\| x(t) - y(t)\| = \epsilon \neq 0$. We derive sufficient synchronization criteria using linear matrix inequality (LMI). Although LMI is a well established method for determining the stability and synchronization threshold for linearly coupled systems, including oscillators with periodic and parametric driving (See for instance [41–46,4–6]), however, previous LMI-based approaches assume a mutual and identical external forcing on the oscillators, which eliminates the effect of periodic forcing on chaos synchronization criteria. We show explicitly that the external forcing parameters, which hitherto has been neglected, plays significant role in the robustness of the criteria.

Furthermore, we also report coupling-induced nonlinear dynamics associated with quasi-synchronization, namely complex bifurcation structures, with co-existing attractors and multiple resonances. The rest of the paper is organized as follows: In the next section, we describe our model system and obtain the synchronization criteria in Section 3. In Section 4, the transition to synchronization is examined. Section 5 is concerned with coupling-induced resonances. The paper is concluded in Section 6 with summary.

2. System description

Most of the models of plasma oscillations are concentrated on unmagnetized and collisionless dusty plasma, made up of electrons, ions and dust grains. They are usually characterized by micron or submicron sized dust particles (See Ref [47], and references therein). In dusty plasma, the dust grain charge is variable, and capable of causing significant modification on the plasma's properties, including its dissipative and nonlinear properties [47,48]. Here, we consider magnetized plasma, whose particle species form separate conducting fluids. The dynamics of this type of plasma has been extensively studied [29–31]. It consists of two interpenetrating fluids of positive ion (i) with charge $+e$ and electron (e) with charge $-e$. The Eulerian equations can be written as: