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# Multifractal regime detecting method for financial time series

Hojin Lee, Woojin Chang *

*Department of Industrial Engineering, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul, Republic of Korea*

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ABSTRACT

We focus on time varying multifractality in time series and introduce a multifractal regime detecting method (MRDM) adopting a nonparametric statistical test for multifractality based on generalized Hurst exponent (GHE). MRDM is a practical method to discriminate multifractal regimes in a time series of any length using a moving time window approach with the adjustable time window size and the moving interval. MRDM is applied to simulations consisting of both multifractal and monofractal regimes, and the results confirm its validity. Using MRDM, we identify multifractal regimes in the time series of Korea composite stock price index (KOSPI) from 1990 through 2012 and observe the distinct stylized facts of the KOSPI return values in multifractal regimes such as the heavy tail distribution, high kurtosis, and the long memory in volatility. Surrogate tests based on improved amplitude adjusted Fourier transformation (IAAFT) algorithm, normal distribution, and generalized student *t* distribution are performed for the validation of MDRM, and the probable causes of multifractality in the KOSPI series are discussed.



## 1. Introduction

Financial time series have various statistical stylized facts which are not expressed using Gaussian models. Some examples are as follow. Returns of financial asset have the fatter tail distribution than the Gaussian noise and show frequent extreme jumps. Volatility of returns is heteroscedastic, long-range dependent, and likely to be clustered. Multifractality is one of these stylized facts observed in many financial time series.

The multifractality is rather a macroscopic concept, but there exist the time ranges showing strong multifractal properties in financial time series. This implies that a time series having multifractality as a whole can be segmented into the multifractal regimes and the non-identifiable regimes inside of the time series in microscopic view. Many financial time series reflecting economic cycles, growth and recession, exhibit the tendency of time-varying multifractality.

Mandelbrot [23] developed a fractal theory measuring the complexity of a fractal structure by defining the fractal dimension distinct from the conventional Euclidean dimension. The fractal structure having multiple dimensions is called "multifractal", while a fractal structure with a single dimension is called "monofractal".

The multifractality in a time series can be observed by the Hurst exponent estimation. Peng et al. [32] and Kantelhardt et al. [16] proposed a multifractal detrended fluctuation analysis (MF-DFA), which can reliably determines the multifractal scaling behavior of nonstationary time series, for the estimation of the exponent. Another method is called the generalized Hurst exponent (GHE) approach. The $q$th order moments of the distribution of increments of time series value are used to estimate the exponent [1,10].

Kantelhardt et al. [16] distinguished two sources of multifractality in time series: the properties of probability density function (PDF) for the values in time series, especially its heavy tail thickness (fat tail) and long range correlation of fluctuations in time series. Kumar and Deo [17] pointed out that both the properties of PDF and long

* Corresponding author. Tel.: +82 2 880 8335.
*E-mail address:* changw@snu.ac.kr (W. Chang).

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range correlation give rise to the multifractality in Indian financial market. While Zunino et al. [43] and Barunik et al. [5] mentioned that heavy tail distribution is the main cause of multifractality, Kwapień et al. [18], Drożdż et al. [12], Oh et al. [28] and Zhou [41] claimed that multifractality in time series is mainly due to the long range correlation property of time series. Drożdż et al. [12] showed that a uncorrelated time series of short length (less than $10^5$ data points) can be misjudged as a multifractal series, and Drożdż et al. [12] and Zhou [41] stated that the properties of PDF have an impact on the multifractality of time series only when the time series possesses long range correlation.

Some overview of a series of research applying scaling property to model financial market is as follows. Bacry et al. [2,3] introduced multifractal random walks (MRW) with stationary increments and continuous dilation invariance. Górski et al. [13] studied the complicated multifractal nature of stock market dynamics. Calvet and Fisher [6–9] have done important work in multifractality in asset returns including volatility forecasting, Markov switching multifractal, and multifractal model of asset returns (MMAR). Lux [21] proposed a generalized method of moments approach for estimating multifractal parameters in Markov-switching multifractal model (MSM). Liu et al. [20] analyzed the multi-scaling properties of financial data using MSM. Oświęcimka et al. [31] focused on the Lux extension to MMAR and applied the model to study the dynamics of the Polish stock market.

Oświęcimka et al. [29] applied MF-DFA to the high frequency stock market data. Kwapień et al. [18] pointed out that nonlinear temporal correlations weigh more than the fat-tailed probability distribution as a contributor to the multifractal dynamics of stock return. Oświęcimka et al. [30] performed a comparative study for the detection of multifractality between MF-DFA and wavelet transform modulus maxima (WTMM) method. Drożdż et al. [12] applied both MF-DFA and WTMM method to detect multifractality in time series, and claimed that the genuine multifractality results from temporal correlation. Zunino et al. [42] applied MF-DFA to developed and emerging stock markets and introduced the multifractality degree to assess stages of stock market development. Zhou [41] decomposed the multifractality into three components caused by linear correlation, nonlinear correlation, and the fat-tailed probability density function (PDF), and maintained that the fat-tailed PDF have an impact on the multifractality with the presence of nonlinear correlation. Kwapień and Drożdż [19] provide a general overview of multifractality in complex systems. Suárez-García and Gómez-Ullate [36] applied MF-DFA to a multifractal and correlation analysis of the high-frequency returns of the IBEX 35 index of Madrid stock exchange.

Among abundant studies on multifractality in financial time series, the research on a statistical multifractality test is relatively small. Wendt and Abry [38] and Wendt et al. [39] suggested bootstrap methods to discriminate multifractality for time series of large sample size ($2^{12}$ or $2^{15}$). However, their methods may not be applicable to the time series of small sample size due to the inaccuracy of resampling. Jiang and Zhou [15] performed statistical tests upon intraday minutely data within individual trading days to check whether the indexes possess multifractality.

Morales et al. [25,26] suggested that financial time series have time varying multifractality. Morales et al. [25] computed weighted generalized Hurst exponent (wGHE) over a moving time window and monitored the dynamics of wGHE during the unstable periods in financial time series. Morales et al. [26] identified the time varying multifractal properties, comparing empirical observations of wGHE with the time series simulated via multifractal random walk by Bacry et al. [3]. Sensoy [33,34] studied the efficiency of stock markets (middle east and north african stock market and federation of Euro-Asian stock exchanges, respectively) using GHE over a moving time window.

In this paper, we introduce a multifractal regime detecting method (MRDM) that identifies multifractal ranges in the time series through a moving time window. By applying MRDM to a time series, we can segment the time series into multifractal and non-identifiable regimes. The multifractality of a time series window is checked and a multifractal regime is detected by rolling the window forward with a regular interval at a time. MRDM adopts the GHE approach considering the time varying multifractal property and a simple nonparametric statistical test to select multifractal regimes. MRDM is appropriate to analyze the time varying multifractality of time series and performs well in the time series of small sample size. MRDM is applied to the simulation of multifractal model of asset returns (MMAR), which is a typical multifractal process, and the empirical data of Korea composite stock price index (KOSPI) ranging from 1990 to 2012.

The remainder of this paper is as follows. In Section 2, stylized facts between the monofractal process and the multifractal process are compared and a multifractality test is introduced and validated using the simulation of monofractal and multifractal processes. In Section 3, MRDM is introduced and its type 1 and type 2 errors are measured based on simulation data consisting of fractional Brownian motion and MMAR. The empirical application of MRDM to the KOSPI series and the related surrogate test results are discussed in Section 4, and the summary and conclusion of this paper are in Section 5.

## 2. Discrimination of multifractality

A time series, $\{X(t)\}$, has the following scale property,

$$E[|\Delta X_\tau(t)|^q] \sim \tau^{\zeta(q)} \tag{1}$$

where $\Delta X_\tau(t) = X(t+\tau) - X(t)$ and $\zeta(q)$ is the scaling function.

The scaling function of fractional Brownian motion (fBm) has a linear form of $\zeta(q) = Hq$, where $H$ is the Hurst exponent, $0 < H < 1$ [27]. Especially, fBm becomes an ordinary Brownian motion when $H = 0.5$. The fBm, $B_H(t)$, is a self similar process and has the following property,

$$B_H(ct) \equiv c^H B_H(t) \quad \text{for} \quad c \geqslant 0 \tag{2}$$

When $H$ is larger than 0.5, fBm is long range dependent.