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One-dimensional map-based neuron model: A logistic modification

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ABSTRACT

A one-dimensional map is proposed for modeling some of the neuronal activities, including different spiking and bursting behaviors. The model is obtained by applying some modifications on the well-known Logistic map and is named the Modified and Confined Logistic (MCL) model. Map-based neuron models are known as phenomenological models and recently, they are widely applied in modeling tasks due to their computational efficacy. Most of discrete map-based models involve two variables representing the slow-fast prototype. There are also some one-dimensional maps, which can replicate some of the neuronal activities. However, the existence of four bifurcation parameters in the MCL model gives rise to reproduction of spiking behavior with control over the frequency of the spikes, and imitation of chaotic and regular bursting responses concurrently. It is also shown that the proposed model has the potential to reproduce more realistic bursting activity by adding a second variable. Moreover the MCL model is able to replicate considerable number of experimentally observed neuronal responses introduced in Izhikevich (2004) [23]. Some analytical and numerical analyses of the MCL model dynamics are presented to explain the emersion of complex dynamics from this one-dimensional map.

1. Introduction

The generic properties of excitable neurons should be accounted in an appropriate neuron model. A suitable neuron model should have a globally attracting equilibrium representing the resting state in the absence of external inputs. In the presence of external stimulation, the excitable neuron can fire and exhibit different oscillations representing spiking and bursting behaviors [1,2]. Thus, in the presence of external stimulation, in addition to the resting state equilibrium, another attractor representing the firing state of the neuron should appear. The firing state attractor is mainly a limit cycle that may bifurcate to a strange attractor for some parameter values. A suitable neuron model also needs sub-threshold and supra-threshold attracting domains. A small stimulation less than the threshold, leaves the system inside the sub-threshold domain and the trajectory of the system would be attracted to the resting equilibrium. In contrast, when the input is large enough to move the system to the supra-threshold domain, the trajectory would be attracted to the firing state attractor [1,2].

Different methods that have been used for modeling these main neuronal characteristics are explained in the rest of this section. Ordinary Differential Equations (ODEs)

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are the conventional tools for modeling the electrical activities of firing neurons started by masterwork of Hodgkin and Huxley in 1952 and since then, quite a few other ODE-based models have been proposed by scientists that are reviewed in [3]. In the meanwhile, by development of digital computers and computer graphics, which considered time as a discrete event, the tendency to use difference equations or maps in modeling tasks has been growing fast [4]. Map-based models have many advantages over continuous-time ODE-based models [5]. Most of the ODE-based neuron models have complex equations with high dimensions that make the analyzing of the collective dynamical mechanisms of a network of channel-based models rather difficult [5]. Thus, a simplified map-based model is a good candidate to explore these dynamical mechanisms [6]. Maps are capable of showing many complex behaviors with less computational cost. Indeed, in the ODE-based models, oscillations and chaotic behaviors need at least two and three dimensions, respectively, whereas even one-dimensional maps are able to produce all of these behaviors [7]. In the recent years, several map-based neuron models have been proposed that most of these models are achieved from discretization of the continuous Integrate-and-Fire (IF) family like Leaky Integrate-and-Fire (LIF) [8] and Quadratic Integrate-and-Fire (QIF) [8,9]. The slow-fast prototype derived from the IF family described in Eq. (1).

$$\begin{cases} x(k+1) = F[x(k), I \pm y(k)] \\ y(k+1) = y(k) \mp \varepsilon[x(k) - qy(k) - \sigma] \end{cases} \tag{1}$$

The Izhikevich, Rulkov [6,10], Courbage-Nekorkin-Vdovin (CNV) [11] and Chialvo [12] models can be named as some examples of these models. The differences between these models are due to different choices of the nonlinear function of the fast dynamic (F) and the parameter values of the slow dynamic (ε, q, and σ) [7].

These models are designed to reproduce the neuronal behaviors, while their parameters do not represent explicit biophysical mechanisms of the neurons. In fact, Map-based models are known as phenomenological models [13]. They are mainly designed based on replicating complex dynamics of neurons rather than modeling the actual mechanisms that generate them [14]. More detailed studies on map-based models and their dynamics are carried on in [5,13].

Although the two-dimensional map-based models have attracted lots of attention in simulation of large neuronal networks, one-dimensional maps are also interesting owing to their less computational complexity. Some one-dimensional maps have been developed to model neuronal behaviors [7]. These models include: the fast subsystem of the chaotic Rulkov model [7,10], the burster model proposed by Cazelles et al. [15], Izhikevich one-dimensional bursters [16], the Nagumo-Sato model [17], and the Aihara model [18].

The first three mentioned models are designed specifically to simulate the neuronal-like behaviors that other conductance-based models with higher dimensions can depict. In the chaotic Rulkov model [10], the slow dynamic can be considered as another parameter in the fast subsystem; therefore, it can be reduced to one-dimensional map [7]. This model is capable of producing chaotic bursting activity, but the bursts does not resemble realistic neuronal bursting behavior [7]. The Cazelles model is a piecewise linear map composed of four linear pieces [15], and is able to exhibit chaotic bursting. The spike amplitude along each burst decreases, which creates Fold/Hopf bursting introduced by Izhikevich [2], but the amplitude and width of bursts are different from each other. Another set of different one-dimensional mappings were introduced by Izhikevich to explain different types of structures involved in bursting (node/focus/fold) that occur in one-dimensional mappings [16].

The Nagumo–Sato is originated from the McCulloch-Pitts [19] approach for artificial neurons rather than from the electrophysiologically motivated Hodgkin–Huxley model [17]. The Aihara model is a modification of the Nagumo-Sato map [18] and the temporal behavior of this model resembles a specific neuronal activity that has been observed experimentally [18].

Each of the above-mentioned one-dimensional map-based models are designed to represent limited aspects of the neuronal dynamics and they do not have the flexibility of showing various kinds of neuronal activities compared to the map-based models with higher dimensions, and ODE-based models [7].

In this paper, a one-dimensional map-based model of complex neural behaviors is developed by applying some modifications to the Logistic map [20]. Logistic map with its simple formula ($x_{n+1} = ax_n(1 - x_n)$), is capable of exhibiting different periodic and chaotic dynamics, and many analytical investigations have been carried out for this map [4,21], which make it a good choice to be the basis of the proposed model. However, the original equation of the Logistic map does not have the ability to display neuronal-like responses. In the present paper, some modifications are applied to the Logistic map that give rise to the formation of various neuronal behaviors like different chaotic and periodic bursting and spiking regimes that will be discussed in details in the rest of this paper.

In the next section, the model formulation is introduced and typical values for the parameters of the proposed model for regular and chaotic behaviors are demonstrated. Temporal responses of the model are represented in Section 3. Section 4 deals with the analysis of the model dynamics and explains the impact of model parameters on the appearance of complex neuronal behaviors and in Section 5, a delicate modification is also offered to make the bursting patterns of the model more realistic. Finally, Section 6 is devoted to conclusion and discussion.

2. The model

The model formulation is given in Eq. (2):

$$x_{n+1} = F(x_n) = \begin{cases} Ax_n^{\alpha}(1 - x_n^{\beta}) + k, & x_n \leqslant 1 \\ 1, & x_n > 1 \end{cases} \tag{2}$$

We call this map the MCL (Modified and Confined Logistic) model. To build this model, three main modifications have been applied to the original Logistic map: