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Multifractal spectrum distribution based on detrending moving average

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ABSTRACT

The time-singularity multifractal spectrum distribution (MFSD) has been proposed recently as a generalized singularity spectrum in a time varying framework. In this paper, we aim at putting forward a new algorithm i.e. MFSD based on detrending moving average (DMA-MFSD) to determine MFSD, which is also a generalization of multifractal detrending moving average (MF-DMA) method. We relate DMA-MFSD method to the MFSD based on the standard partition function, and prove that both approaches are equivalent for fractal time series with compact support. The performance of the DMA-MFSD methods with different moving windows is studied using synthetic fractional Gaussian noise (fGn), binomial multiplicative cascades (BMC) with analytical solutions and real sea clutter data. We find that the estimated DMA-MFSD is in good accordance with the detrended fluctuation analysis based multifractal spectrum distribution (DFA-MFSD) and the theoretical analysis. Overall, the backward DMA-MFSD method has the best performance, which provides the most accurate estimates of the time-singularity MFSD, while the centered DMA-MFSD method performs worse. In addition we find that the backward DMA-MFSD algorithm even outperforms the DFA-MFSD method in the computational complexity and precision.

1. Introduction

Fractals and multifractals are ubiquitous in natural and social sciences since it was presented by Mandelbrot, especially in the field of signal modeling, analysis and processing in the natural world [1], such as electroencephalograms (EEG), electrocardiograms (ECG), as well as turbulent flows [2], seismicity [3], DNA sequences [4], stock market [5], geographical objects [6] and so on. There are a large number of methods developed to characterize the properties of fractals and multifractals, and the classical method is fractal dimensions and multifractal spectrum (MFS) [7]. Multifractal analysis is based on the Hausdorff measure and Hausdorff dimension of fractal subsets, which is extension of fractal dimension. MFS develops the conception of overall fractal dimension, which ignores the local information and details characteristic and is sufficient only for overall depiction of signal. By means of statistical analysis of singularity exponent, MFS describes fractal dimensions of fractal subsets according to different singularity exponent. There are several algorithms for MFS, for example, structure function method, wavelet coefficient based MFS [8], wavelet transform model maxima (WTMM) [9], detrending fluctuation analysis (DFA) and detrending moving average (MF-DMA) [10,11] and wavelet leaders method (WLMF) [12]. These methods focus mainly on estimation accuracy, statistical convergence, limited data length effects, calculation complexity, stability and the mathematical foundation of the MFS [13,14].

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Multifractal singularity spectrum can be regarded as signal representation of fractal signal in singularity domain, which adapts to the determined or stochastic stationary fractal signal. However, when the signal is non-stationary or nonlinear and singular characteristics change over time, e.g. complex turbulent, oscillating singularity signal and geophysics signal, MFS fails to describe singularity spectrum characteristic at any given moment [15,16]. In the past years, the windowed MFS and short-time singularity exponent have been proposed, analogy to the idea of short-time Fourier analysis, to overcome the lack of time information of MFA [15]. Recently, we have proposed the time-singularity multifractal formalism and the *time-singularity multifractal spectrum distribution* (MFSD) based on cyclic autocorrelation function [17] and WTMM method [18], which are based on quadratic wavelet analysis and involve tracing the maxima lines in the wavelet transform over scales. Recently, a multifractal version intended for multiple series is proposed by Zhou [19] as a multifractal version of detrended cross-correlations analysis [20].

In this paper, we develop time-singularity multifractal spectrum distribution based on detrending moving average (DMA-MFSD) as new multifractal time-singularity formalism, which is also a generalization of multifractal detrended moving analysis (MF-DMA) method. The DMA-MFSD method is validated to be equivalent to the MFSD based on standard partition function and statistical advantage in computational complexity, precision and calculating convergence. This method does not require the modulus maxima tracking procedure, and hence does not involve more effort in programming than the conventional MF-DMA. We find that the estimated DMA-MFSD $f(t,\alpha)$ are in good accordance with the detrended fluctuation analysis based multifractal spectrum distribution (DFA-MFSD) and the theoretical analysis. Overall, the backward DMA-MFSD method has the best performance, while the centered DMA-MFSD method performs worse. In addition, we find that the backward DMA-MFSD algorithm even outperforms the DFA-MFSD method and WTMM-MFSD in calculating precision and computational complexity.

In Section 2, the conception of time-singularity multifractal spectrum distribution is introduced, and several algorithms of determination of MFSD are analyzed. The DMA-MFSD is theoretically induced and its algorithm implements are analyzed. We also deduce the relation between DMA-MFSD and standard multifractal spectrum distribution based on the structure function, and prove the uniformity of the two methods in the Section 3. In Section 4, The performance of the DMA-MFSD methods is studied using synthetic fractal and multifractal measures with analytical solutions and real fractal series, including fractional Gaussian noise (fGn), binomial multiplicative cascades (BMC) [21] and real sea clutter [22]. Finally, in Section 5, we give the conclusion.

2. Multifractal spectrum distribution (MFSD)

In this section, we will introduce some conception of time-singularity MFSD [18] that will be used afterwards.

2.1. Time-singularity Hausdorff measure and spectrum distribution

For a function or the path of a process $x(t)$, the instantaneous self-correlation function can be expressed as $r_{xx}(t,\tau) = E[x^*(t+\tau/2)x(t-\tau/2)]$, with the time-delayed conjugation of analyzed signal as the windows function, and then we can obtain the time-varying Holder exponent $h(t,\tau)$ or the time-varying wavelet singularity exponent $W(t,\tau)$. According to anisotropies hypothesis, the statistic character of stochastic process can be obtained by the sampling signal, so the local character of $x(t)$ can be acquired by $h(t,\tau)$ or $W(t,\tau)$. Suppose the $E^{[a]}(t)$ as points subset with same singularity exponent in t, then $E^{[a]}(t) := \{\tau : \liminf_{n\to\infty} h_{kn}^{(n)}(t) = a\}$.

The sets $E^{[a]}(t)(a \in \Re)$ give the multifractal decomposition of signal $x(t)$, i.e. the fractal sets with α structure the support of $x(t)$ at time t. $\dim(E^{[a]}(t))$ reveals the geometry distribution of singularity exponents. In view of

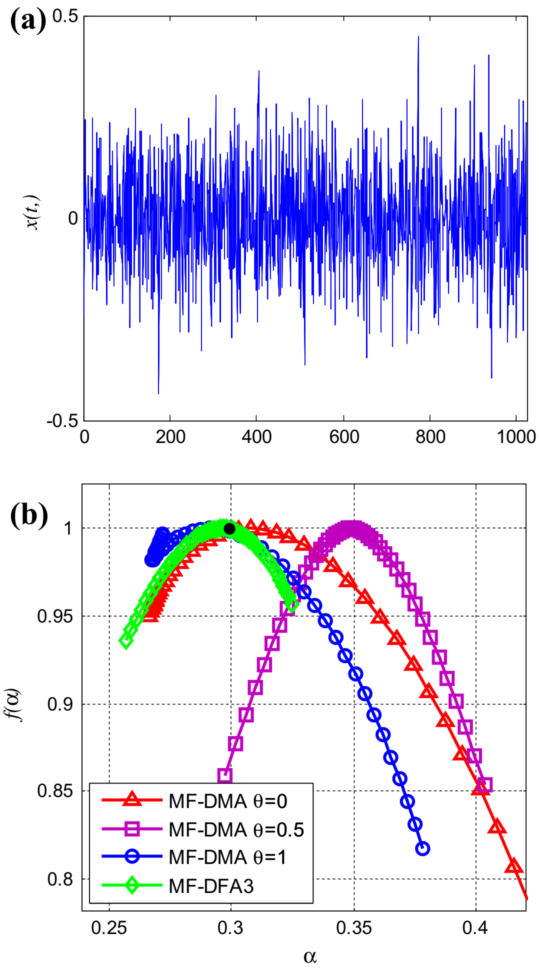

Fig. 1. Fractal Gaussian noise (fGn) and its MF-DMA spectrum. (a) Fractal Gaussian noise with $H = 0.3$, *nbpints* = 2^{10}, (b) the multifractal spectrum of BMC: the symbols 'Δ', '$\Box$', 'O', and '$\Diamond$' with solid line denote backward MF-DMA, centered MF-DMA, forward MF-DMA, and MF-DFA3 methods, respectively; ● denotes the theoretical value.