Physica D 320 (2016) 1–8

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Autoresonance versus localization in weakly coupled oscillators

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HIGHLIGHTS

- An analysis of capture into resonance and escape from it in coupled oscillators under the action of a periodic force is provided.
- An effect of slow modulation of the natural and/or external frequency on the emergence of autoresonance is investigated.
- Explicit asymptotic solutions are derived.
- Numerical simulations prove a good agreement between the analytical and numerical (exact) results.

ARTICLE INFO

Article history:
Received 3 August 2015
Accepted 1 January 2016
Available online 8 January 2016
Communicated by G. Stepan

Keywords:
Nonlinear oscillations
Asymptotic methods
Autoresonance

ABSTRACT

We study formation of *autoresonance* (AR) in a two-degree of freedom oscillator array including a nonlinear (Duffing) oscillator (the actuator) weakly coupled to a linear attachment. Two classes of systems are studied. In the first class of systems, a periodic force with constant (resonance) frequency is applied to a nonlinear oscillator (actuator) with slowly time-decreasing stiffness. In the systems of the second class a nonlinear time-invariant oscillator is subjected to an excitation with slowly increasing frequency. In both cases, the attached linear oscillator and linear coupling are time-invariant, and the system is initially engaged in resonance. This paper demonstrates that in the systems of the first type AR in the nonlinear actuator entails oscillations with growing amplitudes in the linear attachment while in the system of the second type energy transfer from the nonlinear actuator is insufficient to excite high-energy oscillations of the attachment. It is also shown that a slow change of stiffness may enhance the response of the actuator and make it sufficient to support oscillations with growing energy in the attachment even beyond the linear resonance. Explicit asymptotic approximations of the solutions are obtained. Close proximity of the derived approximations to exact (numerical) results is demonstrated.

1. Introduction

It is well known that high-energy resonant oscillations in a linear time-invariant oscillator are generated by an external force whose constant frequency matches the frequency of the oscillator. A change of the forcing and/or oscillator frequency results in escape from resonance. On the contrary, the frequency of a nonlinear oscillator changes as the amplitude changes, and the oscillator may remain in resonance with its drive if the driving frequency and/or other parameters vary slowly in time to be consistent with the slowly changing frequency of the oscillator. The ability of a nonlinear oscillator to stay captured into resonance due to variance of its structural or excitation parameters is termed *autoresonance* (*AR*), or nonstationary resonance.

An idea of "resonance under the action of a force produced by the system's itself" was first suggested by Andronov, Vitt and Khaikin (see [1] for references and details). It was shown [1,2] that the realization of the sought resonance regime in an autonomous system employs feedback control and does not need an additional source of energy. However, feedbacks ensuring stable high-energy oscillations require careful diagnostics of nonlinear states and may become problematic in multi-degree of freedom systems. Proper modulation of structural and/or excitation parameters allows one to excite AR in a system without complicated and costly feedbacks.

AR was first used in applications to particle accelerations [3–5] and planetary dynamics [6–8] and reported as "phase stability principle". Building on that works, a large number of theoretical investigations, experimental results and applications of AR for a broad range of systems have been reported in literature, see, for instance, in [9–16] and references therein. In most of that studies AR in the forced oscillator was considered as an

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http://dx.doi.org/10.1016/j.physd.2016.01.001
0167-2789/

effective tool for exciting a required high-energy regime in the entire system. This paper demonstrates that this conclusion is not common, because capture into resonance of a multi-dimensional oscillator is a much more complicated phenomenon than a similar effect for a single oscillator. As an example, an array consisting of a linear oscillator weakly coupled to a nonlinear (Duffing) actuator driven by an external force is considered in this work. This oscillator cell represents, for example, a model of a micro-energy generator [17]. This model admits a straightforward asymptotic analysis, which allows one to establish the conditions of the occurrence of AR in the entire system and reveal the physical nature of the transient processes.

Two types of excitation are considered in this work: first, the actuator (the Duffing oscillator) with slowly time-decreasing stiffness is excited by a periodic force with constant frequency; secondly, the time-invariant actuator is driven by a force with a slowly increasing frequency. In both cases, the attached linear oscillator and linear coupling remains time-invariant, and the system is initially captured into resonance. The purpose of the present paper is to analyze capture into resonance and escape from it for both types of excitation. Note that the present work does not analyze the global behavior of the system in the phase space. We focus on a particular problem of passage through resonance of an array starting at rest. The last assumption is important, because zero initial conditions correspond to motion with maximum possible energy transfer from a source of energy to a receiver [18,19].

Since multidimensional nonlinear nonstationary systems seldom yield explicit analytical solutions needed for modeling and understanding the transition phenomena, the multiple scales formalism [20] is invoked to derive asymptotic solutions for both types of systems.

It was shown in earlier work [21] that a slow increase of the forcing frequency or an equivalent decrease of the linear stiffness plays a similar role in the emergence and stability of AR in a single Duffing oscillator. However, this analogy becomes invalid for the system under consideration. As shown in Section 2, periodic forcing with constant (resonant) frequency being applied to the nonlinear (Duffing) actuator with slowly-decreasing linear stiffness gives rises to oscillations with growing energy in both oscillators. However, Section 3 proves that if the forcing frequency slowly increases but the actuator is time-invariant, the most part of energy remains localized on the excited oscillator and a portion of energy transferred to the linear oscillator is insufficient to provide growing oscillations in the attachment.

It seems obvious that different dynamical behavior of the two types of system is closely related to their resonance properties. If the system is excited by periodic forcing with constant frequency, both oscillators are captured into resonance: the time-variant nonlinear oscillator remains captured into resonance due to an increase of the amplitude compensating the change of its linear stiffness, while the partial frequency of the linear oscillator is always close to the excitation frequency. If the forcing frequency slowly increases, AR in the time-invariant nonlinear oscillator is still sustained by the growth of the amplitude, while the linear oscillator escapes from resonance. It is important to note that escape from resonance does not prevent further increase of energy of the linear oscillator. The linear oscillator is actually driven by the coupling response with gradually increasing amplitude and thus, the dynamics of the oscillator depends on the relationship between the growth of incoming energy and the loss of energy due to escape from resonance. As shown in Section 3, an additional slow modulation of stiffness can enhance the response of the nonlinear oscillator and make it sufficient to produce oscillations with permanently increasing energy in the linear oscillator even beyond the domain of resonance.

Escape from resonance and formation of bounded oscillations in the entire system is briefly discussed in Section 4. Section 5 contains a brief summary and conclusions.

2. AR in a system subjected to a periodic excitation

2.1. Main equations

In this section we consider a 2DOF model consisting of a linear oscillator with constant parameters weakly coupled to a time-dependent nonlinear (Duffing) oscillator subjected to a periodic excitation with constant frequency. The equations of motion are given by:

$$\begin{aligned} &m_0\frac{d^2u_0}{dt^2}+C(t)u_0+\gamma u_0^3+c_{10}(u_0-u_1)=A\sin\omega t,\\ &m_1\frac{d^2u_1}{dt^2}+c_1u_1+c_{10}(u_1-u_0)=0, \end{aligned} \tag{1}$$

where u_0 and u_1 denote absolute displacements of the nonlinear and linear oscillators, respectively; m_0 and m_1 are their masses; c_1, stiffness of the linear oscillator; c_{10}, the linear coupling coefficient; γ, the coefficient of cubic nonlinearity; $C(t) = c_0 - (k_1 + k_2t)$, where $k_{1,2} > 0$ are detuning parameters; A and ω denote the amplitude and the frequency of the periodic force applied to the nonlinear oscillator (the actuator); the second (linear) oscillator is unforced. The system is initially at rest, i.e., $u_r = 0$, $v_r = \frac{du_r}{dt} = 0$ at $t = 0$ $(r = 0, 1)$. We recall that zero initial conditions define the Limiting Phase Trajectories (LPTs) corresponding to motion with maximum possible energy transfer from a source of energy to a receiver [18,19].

Assuming weak coupling, we define the small parameter ε of the system by relation $\varepsilon = c_{10}/(2c_1) \ll 1$. Then, considering weak nonlinearity and taking into account resonance properties of the system, we introduce the parameters

$$\begin{aligned} &c_0/m_0 = c_1/m_1 = \omega^2, \qquad A/c_0 = 2\varepsilon F,\\ &\quad k_1/c_0 = 2\varepsilon s, \qquad \gamma/c_0 = 8\varepsilon\alpha,\\ &k_2/c_0 = 2\varepsilon^2 s^2\beta\omega, \qquad c_{10}/c_r = 2\varepsilon\lambda_r, \quad r = 0, 1;\\ &\lambda_1 = 1. \end{aligned} \tag{2}$$

Rescaling of (1) according to (2) yield the following equations:

$$\begin{aligned} &\frac{d^2u_0}{d\tau_0^2}+(1-2\varepsilon s\zeta_0(\tau))u_0+2\varepsilon\lambda_0(u_0-u_1)\\ &\quad+8\varepsilon\alpha u_0^3 = 2\varepsilon F\sin\tau_0,\\ &\frac{d^2u_1}{d\tau_0^2}+u_1+2\varepsilon\lambda_1(u_1-u_0)=0, \end{aligned} \tag{3}$$

where $\tau_0 = \omega t$ and $\tau = \varepsilon s\tau_0$ denote the dimensionless fast and slow time scales, respectively; $\zeta_0(\tau) = 1 + \beta\tau$. Eqs. (3) can be asymptotically analyzed with the help of the multiple time scale method [20]. To this end, we define the complex amplitudes Ψ_r, Ψ_r^* and additional rescaled parameters by formulas:

$$\begin{aligned} &\Psi_r = \Lambda^{-1}(v_r + iu_r)e^{-i\tau_0}, \qquad \Psi_r^* = \Lambda^{-1}(v_r - iu_r)e^{i\tau_0},\\ &\Lambda = (s/3\alpha)^{1/2}, \qquad f = F/s\Lambda,\\ &\mu_r = \lambda_r/s; \quad r = 0, 1; \qquad \mu_1 = s^{-1}. \end{aligned} \tag{4}$$

It follows from (3), (4) that energy E_r of each of the oscillators (3) can be asymptotically evaluated as $E_r = 1/2|\Psi_r|^2 + \varepsilon \ldots$.

Substituting (4) into (3), we obtain the following (still exact) equations in the standard form for the complex amplitudes Ψ_0, Ψ_1:

$$\begin{aligned} &\frac{d\Psi_0}{d\tau_0} = -i\varepsilon s[(\zeta_0(\tau) - |\Psi_0|^2)\Psi_0 - \mu_0(\Psi_0 - \Psi_1) + f + G_0],\\ &\frac{d\Psi_1}{d\tau_0} = i\varepsilon s[\mu_1(\Psi_1 - \Psi_0) + G_1], \end{aligned} \tag{5}$$

with initial conditions $\Psi_0(0) = \Psi_1(0) = 0$. By construction, the functions G_0 and G_1 include the sums of fast harmonics with