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Arbitrary bending of optical solitonic beam regulated by boundary excitations in a doped resonant medium

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HIGHLIGHTS

- Borrowing ideas from optical communication we propose arbitrary bending of light.
- Nonlinear Kerr medium coupled to Erbium doped resonant medium is considered.
- Coupled NLS–SIT as governing equations allows exact accelerating solitonic beam.
- Population inversion of the boundary dopant atoms controls the beam bending.
- 2D and 3D optical beam can bend beyond parabolic and circular curves obtained so far.

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ABSTRACT

Bending of a shape-invariant optical beam is achieved so far along parabolic or circular curves. Borrowing ideas used in nonlinear optical communication, we propose such a bending along any preassigned curve or surface, controlled by the boundary population inversion of atoms in an Erbium doped medium. The optical beam generated in a nonlinear Kerr medium and transmitted through a doped resonant medium preserving its shape as an accelerating soliton, predicted here based on exact solutions, should be realizable experimentally and applicable to nonlinear events in other areas like plasma or ocean wave.

1. Introduction

1.1. Bending of light beam

It is our long-standing dream, that it would be possible some day to produce a localized light beam in a medium, that could be bent to any preassigned curve maintaining its shape throughout the propagation. Such a self-accelerating beam would be able to self-bend around an obstacle, making it virtually invisible. However, the first step in achieving the bending of an optical beam through theoretical and experimental studies was taken quite late [1,2], inspired by a pioneering work of Berry et al. related to accelerating free quantum particles [3]. The beam, typically an Airy function solution of linear Schrödinger equation, could preserve its parabolic form over a finite distance. Subsequently, acceleration along an arbitrary curve was achieved, though at the cost of non-preservation of the shape [4]. Meanwhile bending of propagating solitons has been achieved in other fields, e.g. ring accelerator for matter-wave solitons [5], in chirped photonic lattices [6], self-bending of soliton in optical lattices [7], etc.

Bending of an optical beam along a circular path is reported recently for a shape-sustaining optical beam on a plane, breathing periodically in time, as a Bessel function related exact solution to the linear Maxwell equation in vacuum [8].

A parallel development has taken place involving nonlinear media, in which an Airy-like beam solution was found to the nonlinear Schrödinger (NLS) equation in a Kerr medium [9,10]. Negating the prevailing skepticism, that symmetric nonlinearities cannot produce self-accelerating soliton, a shape-invariant static beam oriented along a parabolic curve on a plane was obtained

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and generalized also to 3D, which subsequently has been verified in real experiments [11,10].

Nevertheless, in all these exciting achievements, the bending of a shape-preserving beam is restricted basically to two specific curves: circular or parabolic, linked to the Airy or Bessel function solutions [1–3,8–10]. This situation however falls way short of our expectation of having a stable optical beam that could bend along any desired curve, regulated by an arrangement in the medium and turn any object invisible by bending around it. We offer here a breakthrough by proposing a scenario close to the above expectation, where a localized 2D or 3D optical beam could be accelerated arbitrarily, resulting in its bending along an arbitrary curved path. The nature of the curve can be controlled by the boundary configuration of the population inversion of atoms in a doped resonant medium, through which the shape-invariant beam passes. Therefore, the optical beam can turn around an object of arbitrary shape without hitting it. The idea of our construction for the bending of an optical beam follows a path, fundamentally different from those adopted so far, as briefed above, and is inspired by a series of earlier studies in the nonlinear optical communication [12–20]. Though our proposal is based on theoretical models we hope it is feasible through future experiments.

1.2. Solitonic optical communication

Soliton based optical communication rests on the principle that, when the intensity of the optical pulses crosses a threshold value the fiber medium behaves nonlinearly, since the refractive index becomes intensity dependent. Moreover, the self-phase modulation (SPM) due to Kerr nonlinearity acts in opposition to the dispersion in the medium at the anomalous dispersion regime. At proper choice of the parametric set the effect of dispersion could be at perfect balance with that of the nonlinear SPM creating a condition for the stable solitonic pulse propagation of the electric field $E(z,t)$, with its dynamics governed by the NLS equation

$$iE_z + E_{tt} + 2|E|^2E = 0. \tag{1}$$

In deriving the well known and well studied integrable NLS equation (1) in $(1+1)$ dimensions from the $(3+1)$ dimensional Maxwell equation in a nonlinear medium, among many simplifying assumptions a key requirement is the separation of variables for the electric field:

$$\mathbf{E}(\mathbf{r},t) = \frac{\hat{\mathbf{x}}}{2}(E(z,t)F(x,y)e^{i(k_0z-\omega_0t)} + \text{cc.}), \tag{2}$$

with proper choice of $k_0(\omega_0)$, where $E(z,t)$ is the slowly varying envelope of short pulses with an assumption of a slower variation along z, such that the second derivative of the field E_{zz} can be neglected [21]. Note that, fiber communication through an electric field, transmitted in a Kerr medium as a stable solitonic pulse moving with a constant velocity, was proposed long ago [12].

Another proposal, which gained popularity for improved transmission of nonlinear optical pulses, is due to self-induced transparency (SIT) produced by the coherent response of a medium to ultra-short pulses [13]. The medium is doped usually with Erbium Er^{3+} ions, since this rare earth material prominently exhibits an effective two-level energy spectrum (with $^4I_{\frac{15}{2}}$, as the ground state and $^4I_{\frac{13}{2}}$, as the excited state), though with a smearing of the levels due to the Stark effect. Moreover, the energy difference between these two levels is nearly equal to that of the wavelength at which soliton pulses are propagated. Therefore the resonant interaction makes the two-level medium optically transparent to that wavelength, inducing a SIT process. In fact, the interaction of the fields, involving polarization p in the resonant medium induced by the propagating optical field E and population inversion (PI) N of an ensemble of two-level atoms, is responsible for making the resonant medium transparent to the wavelength at which the optical pulses are transmitted. Amplification and long-distance soliton transmission is now successfully achieved for practical purposes using distributed Er^{3+}-doped fibers [14–16]. The dynamics of the fields is described by the SIT equations

$$iE_z = 2p, \qquad ip_t = 2(NE - \omega_0 p), \qquad iN_t = E^*p - p^*E, \tag{3}$$

which are reduced from the Maxwell–Bloch (MB) equations through an ensemble average together with assumptions of a sharp resonance and homogeneous broadening [13,17,18] and an equally essential requirement of separation of variables for the electric field $E(z,t)$ like (2), together with a similar assumption for the polarization $p(z,t)$ and the PI $N(z,t)$. The SIT equations represent also an integrable system generating solitonic pulses moving with a constant velocity [13].

A subsequent important development is to combine these two models by suitably fabricating the optical fibers for coherent transmission of optical pulses. When a two-level resonant medium such as Erbium is doped with the core of the optical fiber the propagation of optical soliton is described by the reduced MB equations (3), whereas the silica material of the fiber induces the solitonic propagation due to NLS equation (1). Therefore the wave propagation can have both these effects due to silica and doped medium, with Er^{+3}-impurities accounting for the SIT effect, while the silica material gives the NLS soliton effect. The dynamics of the combined system, with wave propagation in Erbium-doped optical fibers is governed by the coupled NLS–SIT equations

$$iE_z + E_{tt} + 2|E|^2E = 2p, \tag{4}$$

$$ip_t = 2(NE - \omega_0 p), \qquad iN_t = E^*p - p^*E, \tag{5}$$

where again the equations can be reduced from the $(3+1)$ dimensional Maxwell and Bloch equations through separation of variables like (2) for the slowly varying envelope and with similar assumption for p, N. In this NLS–SIT system an optical soliton, created in a Kerr medium, is transmitted through an Erbium doped resonant medium inducing polarization and excitation of the two-level dopant atoms, which has been theoretically predicted first by Maimistov and Manyakin [17] and since then has been developed and experimentally verified [18,19], extended further [22,20] and implemented for practical usage [14–16]. Further details on this combined system can be found in a recent monograph [23]. Since the ensemble of dopant atoms in the medium acts as an effective two-level system with the energy gap between the levels being close to that of the wavelength of the propagating solitonic pulses, the resonant interaction induces a balance between the optical absorption and emission, sustaining a stable shape-preserving pulse propagation. Remarkably, this coupled system turns out also to be integrable admitting exact soliton solutions for all fields propagating with the same constant velocity [18].

However, since the SIT process becomes effective for ultra-short temporal pulses, coupling it with the NLS equation might require consideration of the higher-order dispersion in time, modifying the NLS equation to its higher order, known as the Hirota equation:

$$iE_z + E_{tt} + 2|E|^2E + i\alpha(E_{ttt} + 6|E|^2E_t) = 2p, \tag{6}$$

coupled to the same SIT equation (5). It has been shown in [22], that the new set of Hirota–SIT equation is also integrable, admitting a similar soliton solution moving with a constant but higher velocity.