Physica D 272 (2014) 39–50

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Theoretical study on the translation and rotation of an elliptic camphor particle

Keita Iida [a], Hiroyuki Kitahata [b,c], Masaharu Nagayama [d,e,*]

[a] Graduate School of Medicine, Tohoku University, 2-1, Seiryo-machi, Aoba-ku, Sendai, 980-8575, Japan
[b] Graduate School of Science, Chiba University, 1-33, Yayoi-cho, Inage-ku, Chiba, 263-8522, Japan
[c] PRESTO, Japan Science and Technology Agency, 4-1-8, Honcho Kawaguchi, Saitama, 332-0012, Japan
[d] Research Institute for Electronic Science, Hokkaido University, N20W10, Kita-Ward, Sapporo, 001-0020, Japan
[e] CREST, Japan Science and Technology Agency, 4-1-8, Honcho Kawaguchi, Saitama, 332-0012, Japan

HIGHLIGHTS

- We model the spontaneous motion of an elliptic camphor particle atop water.
- Numerical computation clarified the solution structure of the mathematical model.
- The bifurcation points of travelling and rotating solutions are calculated.
- We find that the elliptic camphor particle tends to move in the short-axis direction.

ARTICLE INFO

Article history:
Received 2 May 2013
Received in revised form
13 December 2013
Accepted 24 January 2014
Available online 31 January 2014
Communicated by Y. Nishiura

Keywords:
Surfactant–water system
Reaction–diffusion system
Bifurcation theory
Perturbation theory

ABSTRACT

The spontaneous motion of an elliptic camphor particle floating on water is studied theoretically and experimentally. Considering a mathematical model for the motion of an elliptic camphor particle in a two-dimensional space, we first investigate the asymptotic solutions with numerical computation. We then introduce a small parameter ε into the definition of the particle shape, which represents an elliptic deformation from a circular shape and, by means of perturbation theory, we analytically calculate the travelling solution to within $O(\varepsilon)$. The results show that short-axis-directed travelling solutions primarily bifurcate from stationary solutions and that long-axis-directed ones are secondary which means that elliptic camphor particles are easier to move in the short-axis direction. Furthermore, we show that rotating solutions bifurcate from stationary solutions and that the bifurcation point changes with $O(\varepsilon^2)$, which suggests that elliptic camphor disks easily exhibit translational motion, rather than rotational, within the small deformation. Finally, our theoretical suggestions are confirmed by an experiment.

1. Introduction

Spontaneous motion seen in the natural world has been an origin of fascination for many researchers. What attracts us are the various manners of these motions and relevant phenomena, such as spatiotemporal patterning. Examples include locomotion of motor proteins in living organisms [1], swimming motions of motile bacteria [2], the chemical locomotion of metallic nanorods in aqueous solutions [3], the spontaneous motion of surfactant particles atop liquid surfaces [4–6], spontaneous droplet motions [7–10] and propagating solitary waves in a vertically vibrating suspension [11].

To create a novel chemical motor that works efficiently under isothermal conditions, scientists have been taking a cue from biological motors which convert chemical energy into mechanical energy under almost isothermal and non-equilibrium conditions [12]. A so-called *surfactant–water* system has been introduced as a prototype of such a chemo-mechanical energy transducer. For example, an oil–water system in which an oil droplet exhibits vectorial motion in an aqueous solution was investigated in 1993 and 1996 [7,12]. In this system, the interaction of the surfactant molecules in the aqueous solution and the iodide ions in the oil droplet imparts a surface instability on the droplet, by means of

* Corresponding author at: Research Institute for Electronic Science, Hokkaido University, N20W10, Kita-Ward, Sapporo, 001-0020, Japan. Tel.: +81 11 706 3357; fax: +81 11 706 2891.
E-mail address: nagayama@es.hokudai.ac.jp (M. Nagayama).

http://dx.doi.org/10.1016/j.physd.2014.01.005
0167-2789/

a surfactant molecular layer which changes the surface pressure. As a result, spontaneous motion and deformations are observed.

On the other hand, a camphor–water system in which a camphor particle exhibits unidirectional motions, such as rotation and translation, atop water was introduced [4]. Regarding this system, both experimental and theoretical studies showed that the spontaneous motion of surfactant particles is attributable to nonuniformity of surface tension around the particle [13,14]. They also provided an important idea for realizing such nonuniformity; that is, the asymmetric design of the system, e.g., shape of surfactant particles, shape of the chamber, chemical concentration field, etc. Lately, a number of studies have given a variety of examples, along with improved understanding of spontaneous motions [8,9,15–18].

Considering the simple case in which a surfactant particle or droplet moves over an aqueous surface, one can see that an asymmetric surface tension profile is necessary for inducing motion. In fact, theoretical studies of a camphor–water system have shown that the symmetric profile of surface tension for a standing camphor disk can be unstable with respect to an infinitesimal perturbation, and then the motion in a certain direction is stabilized [14,19]. Consequently, the surface concentration of camphor molecules becomes asymmetric with respect to the center of the camphor disk.

In the last decade, motions of surfactant particles, almost undeformable droplets, and that of "camphor boats" as well, have been investigated by means of mathematical modeling under certain conditions [14,20,21]. As mentioned above, they can explain the spontaneous symmetry breaking and many other phenomena, such as oscillatory motions in a linear chamber [13] and jamming in an annular channel [22–24]. Nevertheless, there are still many problems which are difficult to explain using their spatial one-dimensional model. One example is found in an alcohol–water system, in which an alcohol droplet with an appropriate volume deforms spontaneously due to the Marangoni effect and then exhibits vectorial motion over an aqueous surface [9]. The deformation of the droplet seems to drive the motion and vice versa, but the relationship between motion and deformation has not yet been fully understood. Another example is a translational motion coupled with rotation in a camphor–water system whose spatial two-dimensional model was introduced in 2005 [25]. However, so far, this model has been analyzed only for a circular disk, which cannot exhibit rotational motion [19].

Among these problems, we are interested in the relationship between the shape (or deformation) of surfactant particles (or droplets) and the spontaneous motion. One possible approach is to investigate the shape-dependent motion neglecting the dynamics of deformation. For this purpose, we consider the camphor–water system. The definition of the particle shape is based on recent research on the self-propelled motion of deformable domains. As in [26,27], small deformation of a circular shape with a radius r_0 can be written as

$$r(\theta) = r_0\left[1 + \sum_{k=2}^{\infty}(a_k \cos k\theta + b_k \sin k\theta)\right], \tag{1}$$

where (r, θ) are two-dimensional polar coordinates, and a_k and b_k are infinitesimally small parameters. Here, we choose the 2-mode deformation by dropping all k-mode terms except for $k = 2$. This is the most fundamental, but nontrivial, deformation that is an elliptic deformation.

When considering the elliptic shape of a camphor particle, we face a simple question: in which direction does the elliptic camphor particle move? The present study provides us with an answer to this question by means of numerical and analytical investigations based on mathematical modeling, as well as on experimental observations.

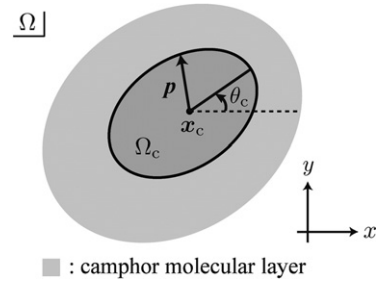

Fig. 1. Schematic representation of the camphor–water system in a two-dimensional space.

2. Mathematical model

Based on previous papers [14,25,28], we introduce a mathematical model for the spontaneous motion of a camphor particle atop a water surface. Let us consider a two-dimensional space Ω $(=\mathbb{R}^2)$ as the water surface and its subset Ω_c as the interior of the camphor particle whose boundary is defined as follows:

$$\partial\Omega_c(\boldsymbol{x}_c, \theta_c) = \{\boldsymbol{x} \in \Omega \mid \boldsymbol{x} = \boldsymbol{x}_c + \boldsymbol{p}\}, \tag{2}$$

where $\boldsymbol{x}_c(t)$ $(=(x_c(t), y_c(t)))$ and $\theta_c(t)$ are the center of mass and characteristic angle of the camphor particle, respectively, and $\boldsymbol{p}$ is the parametric representation for $\partial\Omega_c$ (see Fig. 1). Let $u(t, \boldsymbol{x})$ be the surface concentration of the camphor molecular layer.

The model equations are as follows:

$$m\frac{d^2\boldsymbol{x}_c}{dt^2} = -\eta_t\frac{d\boldsymbol{x}_c}{dt} + \boldsymbol{F}, \tag{3}$$

$$I\frac{d^2\theta_c}{dt^2} = -\eta_r\frac{d\theta_c}{dt} + T, \tag{4}$$

$$\frac{\partial u}{\partial t} = D\Delta u - \alpha u + f(\boldsymbol{x}, \boldsymbol{x}_c, \theta_c), \tag{5}$$

where m and I are the mass and moment of inertia of the camphor particle, η_t and η_r are the friction coefficients for translation and rotation, and D and α are the diffusion coefficient and sublimation rate of the camphor molecular layer, respectively. In (3) and (4), $\boldsymbol{F}$ and T are the driving force and torque exerted on the camphor particle due to the surface tension profile on $\partial\Omega_c$, which are defined as follows:

$$\boldsymbol{F} = \int_{\partial\Omega_c} \gamma(u)\boldsymbol{n}\,d\ell, \tag{6}$$

$$T = \int_{\partial\Omega_c} \gamma(u)\boldsymbol{p} \times \boldsymbol{n}\,d\ell, \tag{7}$$

where $\boldsymbol{n}$ is the outer unit normal vector on $\partial\Omega_c$. The notation "$\times$" denotes the two-dimensional vector product. Surface tension γ is assumed to be a decreasing function of u. Based on the experimental measurements [29], we apply the Hill-type function as follows:

$$\gamma(u) = \frac{\beta^n\gamma_0}{\beta^n + u^n} + \gamma_1, \tag{8}$$

where β, γ_0 and γ_1 are positive constants and n is a positive integer. In (5), f is given by

$$f(\boldsymbol{x}, \boldsymbol{x}_c, \theta_c) = \begin{cases} f_0, & \boldsymbol{x} \in \bar{\Omega}_c, \\ 0, & \boldsymbol{x} \in \Omega \setminus \bar{\Omega}_c, \end{cases} \tag{9}$$

where f_0 is the supply rate of the camphor molecular layer and $\bar{\Omega}_c$ is the closure of Ω_c. The initial conditions are

$$\left(\boldsymbol{x}_c(0), \frac{d\boldsymbol{x}_c}{dt}(0)\right) = (\boldsymbol{x}_0, \boldsymbol{v}_0), \tag{10}$$