Physica D 268 (2014) 91–99

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Inertial focusing of small particles in wavy channels: Asymptotic analysis at weak particle inertia

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HIGHLIGHTS

- The motion of particles in a channel with periodic corrugations is analyzed.
- Some particles focus towards an attracting streamline due to their inertia.
- Little is known about the exact conditions under which this phenomenon occurs.
- We present an asymptotic description of this problem, and solve it.
- These analytical results are confirmed by numerical simulations.

ARTICLE INFO

Article history:
Received 20 February 2013
Received in revised form
17 September 2013
Accepted 5 November 2013
Available online 15 November 2013
Communicated by J. Dawes

Keywords:
Inertial particles
Attractor
Asymptotic method

ABSTRACT

The motion of tiny non-Brownian inertial particles in a two-dimensional channel flow with periodic corrugations is investigated analytically, to determine the trapping rate as well as the exact position of the attractor, and understand the conditions under which particle trapping and long-term suspension occur. This phenomenon has been observed numerically in previous works and happens under the combined effects of confinement and inertia. Starting from the particle motion equations, a Poincaré map is constructed analytically in the limit of weak inertia and weak channel corrugations. It enables to derive the equation of the attractor, if any, and the corresponding trapping rate. The attractor is close to a streamline, the so-called "attracting streamline", and is shown to persist in the presence of transverse gravity, provided the channel Froude number is large enough. Particles which are trapped by this streamline can therefore travel over long distances, avoiding deposition. Numerical simulations confirm the theoretical results at small particle response times τ and reveal some non-linear effects at larger τ: the asymptotic attractor becomes unstable at some critical value and splits into multiple branches each with its own basin of attraction.

1. Introduction

Small inertial non-Brownian particles (like aerosols, dust, sediments or droplets) carried by a spatially-periodic incompressible flow are often observed to focus towards some preferential trajectories (see for example Maxey [1], Maxey and Corrsin [2], Fernandez de La Mora [3], Robinson [4] to name but a few). In terms of dynamical systems, this phenomenon is due to the fact that, under the effect of their inertia, particles have a dissipative dynamics within a bounded phase space, and eventually converge to some attractor with zero volume. The dynamics of inertial particles is therefore very different from the one of pure tracers since the latter have non-dissipative dynamics in volume-preserving flows. In particular, the existence of an attractor significantly affects collisions and aggregation phenomena (for a review see for example Cartwright et al. [5]). These features, and more generally the fact that inertial particles do not follow fluid points, have been observed numerically and experimentally in both laminar and turbulent flows in the past decades (Squire and Eaton [6], Maxey [1], Maxey and Corrsin [2], Haller and Sapsis [7], Qureshi et al. [8], Ijzermans et al. [9], Wilkinson and Mehlig [10], Wilkinson et al. [11]). Recent developments in inertial microfluidics (see for example Di Carlo [12]) have also revived an interest in particle focusing and separation in confined flows. However, in microfluidic applications focusing is usually attributed to the transverse (lift) force due to the proper inertia of the fluid rather than that of the inclusion, which brings them out of the scope of this paper.

In this paper, we study the influence of inertial focusing on particle transport in a two-dimensional smooth channel with corrugated walls. This flow is of great interest in geoscience,

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0167-2789/$ – see front matter
http://dx.doi.org/10.1016/j.physd.2013.11.001

as it can be used to model sediment transport in fractures or mineral separation devices. Wall corrugation imposes curved fluid streamlines inside the channel, and it has been shown numerically that this can cause some particles to focus towards some preferential trajectory or "attracting streamline" (Dahneke [13], Fernandez de la Mora [3]) located somewhere between the two walls. Such particles are sometime referred to as "trapped", since they are forced to move along a well-defined trajectory and cannot escape it. As will be shown below, this phenomenon persists in the presence of gravity, and this can significantly change particle transport properties in channels: particles traveling on the attracting streamline, avoiding sedimentation, can travel over long distances whereas other particles will be eventually deposited on the walls. In most analyses dealing with this phenomenon, a complete analytical description of the attractor is missing, the exact position of the attracting streamline is unknown, and a general bifurcation diagram (indicating under which conditions trapping occurs) is not available. The present paper fills this gap by means of a rigorous asymptotic approach. Indeed, the complex behavior of particle dynamics in fractures results from the combined effect of the fracture shape, particle inertia, and of the various hydrodynamic forces. All these effects contribute to the final focusing rate, and an analytical treatment, like the one presented here, is the only way to deeply understand the relative contribution of these various physical mechanisms. The goal of this paper is therefore to find, as analytically as possible, the explicit conditions for the existence of an attracting streamline in a corrugated channel, and to obtain a complete trapping diagram in terms of flow rate, gravity and channel geometry. In addition, relevant and numerically costly statistical quantities, like deposition lengths or percentage of deposited particles, will also be calculated. Because the inertia of the fluid and wall effects will be neglected in this work, we do not expect our results to match all realistic situations, but the model will help us to understand some key features of inertial particle focusing.

In the following sections (except in Section 6.3), the influence of inertia on the particle motion will be assumed to be weak so that, in the long term, particle velocities will be obtained by using a perturbation of the fluid velocity field. The inertia of the fluid flowing through the channel will also be neglected, and we will use the LCL (Local Cubic Law) flow model [14]. Under these assumptions we study the migration of particles across a periodic corrugated channel (Section 3), and construct the Poincaré map of particle positions at the end of each corrugation period. The use of this map reduces the problem to the calculation of a migration function $f(\eta)$, which contains all information about the long-term behavior of inertial particles. We then apply it to the LCL flow and obtain the trapping diagram which predicts focusing and sedimentation regimes depending on the Froude number and on the channel geometry. Asymptotic estimates of the focusing rate, the percentage of permanently suspended particles and deposition lengths are also calculated. These analytical results are then compared to numerical simulations in Section 6.

2. Particle motion equations and asymptotic expansion

We consider a fluid flow within a two-dimensional corrugated channel (Fig. 1). The typical fluid velocity is U_0 and the scale of corrugations is denoted as L_0. The typical gap of the channel is H_0, and we set $\varepsilon = H_0/L_0$. The channel is taken to be horizontal, being understood that the results presented in this paper can be easily generalized to channels with an oblique direction. Particles are taken to be much smaller than the gap of the channel, and are assumed to move far from the wall. Their density is comparable to that of the fluid. In addition, we assume that they are non-Brownian, that they do not interact and do not modify the main flow. By using U_0 and L_0 to non-dimensionalize the Maxey–Riley equations (in the form derived by Maxey and Riley [15] with corrected added mass [16]) we get:

$$
\begin{aligned}
\frac{d\vec{x}_p}{dt} &= \vec{v}_p, \\
\frac{d\vec{v}_p}{dt} &= -\frac{1}{\tau}\left(\vec{v}_s - \frac{\chi_L^2}{6}\nabla^2\vec{u}_f\right) + \left(1 - \frac{3R}{2}\right)\vec{G} \\
&\quad + \frac{3R}{2}\frac{D}{Dt}\left(\vec{u}_f - \frac{\chi_L^2}{20}\nabla^2\vec{u}_f\right) \\
&\quad + \frac{3\sqrt{R}}{\sqrt{2}\sqrt{\tau}}\int_{-\infty}^{t}\frac{1}{\pi(\sqrt{t-s})}\frac{d}{ds}\left(\vec{v}_s - \frac{\chi_L^2}{6}\nabla^2\vec{u}_f\right)ds,
\end{aligned}
\tag{1}
$$

where $\vec{v}_p$ and $\vec{u}_f$ are the particle and fluid velocities respectively, $\vec{v}_s = \vec{v}_p - \vec{u}_f(\vec{x}_p)$ is the slip velocity at the particle position $\vec{x}_p$. Also, $\chi_L = a/L_0$ is the non-dimensional particle radius and $R = 2/(2\rho + 1)$, where ρ is the particle to fluid density ratio. The non-dimensional gravity vector is $\vec{G} = \vec{e}_z/\mathrm{Fr}$, where $\mathrm{Fr} = U_0^2/g\,L_0$ is the Froude number measuring inertia forces compared to the force of gravity (g is the acceleration of gravity). The main parameter measuring particle inertia is the non-dimensional response time $\tau = \mathrm{St}/R$, where St is the particle Stokes number which can be written as $\mathrm{St} = \frac{2}{9}\mathrm{Re}_L\chi_L^2$ with $\mathrm{Re}_L = U_0L_0/\nu$. In terms of ε we also have $\mathrm{St} = \frac{2}{9}\varepsilon\mathrm{Re}_H\chi_H^2$, with $\chi_H = a/H_0$ being the particle size related to the mean gap of the channel. The motion equation takes into account the following forces (in order of appearance): the drag, the gravity and buoyancy forces, the pressure gradient of the unperturbed flow, the added mass force, and the Basset force.

The terms proportional to the Laplacian of the velocity field are usually referred to as Faxen corrections [17] and are due to the local non-uniformity of the fluid flow at the particle scale. They are very often neglected under assumption that the particle size a is small compared to the typical length scale of the flow L_0. However, even if these terms are small compared to the fluid velocity, they can compete against the particle slip velocity, especially in weakly-inertial regime. The integral term is due to the unsteadiness of the disturbance flow due to the particle. In this work we will assume this disturbance flow is quasi-steady, so that the Basset force will be neglected.

When the particle response time is small ($\tau \ll 1$), the inclusion quickly forgets its initial condition and, in the long term, its velocity does not deviate much from the one of a fluid point (weakly-inertial regime). When $\tau \gg 1$ the particle motion is mostly governed by gravity or other external forces (ballistic regime). In the intermediate case $\tau = O(1)$ the particle dynamics can be very complex. Because we consider particles that are much smaller than the gap of the channel ($\chi_H \ll 1$), the response time $\tau = \frac{2}{9}\frac{(\varepsilon\mathrm{Re}_H)\chi_H^2}{R}$ will always be small unless $R \ll 1$ or $\varepsilon\mathrm{Re}_H \gg 1$. In our study the density ratio R will vary from $R \ll 1$ (heavy particles) to $R = 2/3$ (neutrally buoyant particles) and $R > 2/3$ (particles lighter than the fluid), but we assume that τ is small even for very heavy particles. Throughout the paper we focus on the asymptotic regime at $\tau \ll 1$, except in Section 6.3 where finite response times will be considered.

The motion equations (1) are singularly perturbed in the limit $\tau \to 0$. Physically, this means that the system has two time scales: the hydrodynamic time $T_0 = L_0/U_0$ and the particle response time $\tau\,T_0$. It is known [1,18] that, for small τ, solutions of this equation will converge at an exponential rate $\exp(-t/\tau)$ towards the solutions of the following first-order equation:

$$
\begin{aligned}
\dot{\vec{x}}_p &= \vec{v}_\tau(\vec{x}_p), \\
\vec{v}_\tau(\vec{x}_p) &= \vec{u}_f(\vec{x}_p) + \tau\vec{v}_1(\vec{x}_p) + \tau^{3/2}\vec{v}_{3/2}(\vec{x}_p) + O(\tau^2),
\end{aligned}
\tag{2}
$$