Physica D 239 (2010) 1214–1224

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Nonlinear hydrodynamic phenomena in Stokes flow regime

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ARTICLE INFO

Article history:
Available online 16 December 2009

Keywords:
Stokes flow
Drops
Suspensions
Chaos
Structural transitions
Collective dynamics

ABSTRACT

We investigate nonlinear phenomena in dispersed two-phase systems under creeping-flow conditions. We consider nonlinear evolution of a single deformed drop and collective dynamics of arrays of hydrodynamically coupled particles. To explore physical mechanisms of system instabilities, chaotic drop evolution, and structural transitions in particle arrays we use simple models, such as small-deformation equations and effective-medium theory. We find numerical and analytical solutions of the simplified governing equations. The small-deformation equations for drop dynamics are analyzed using results of dynamical systems theory. Our investigations shed new light on the dynamics of complex fluids, where the nonlinearity often stems from the evolving boundary conditions in Stokes flow.

1. Introduction

The Navier–Stokes equations contain the inertial term that gives rise to numerous nonlinear phenomena, such as flow instabilities [1], complex convective patterns [2], and turbulence [3]. However, there also exist nonlinear hydrodynamic phenomena that are not due to nonlinear inertial contributions. These nonlinear phenomena occur under creeping-flow conditions in interfacial and particulate flows. The Stokes equations governing the fluid flow are linear so the nonlinearity stems entirely from the evolving boundary conditions.

We present two examples of multiphase systems that exhibit complex nonlinear behavior under creeping-flow conditions. The first system is a deformable highly viscous drop subject to external 2D linear flow. The second example is an ordered array of rigid spherical particles in strongly confined Poiseuille flow. The nonlinear coupling in the first system results from the influence of the external flow on the shape of the deformed drop. The nonlinearity in the other system stems from the hydrodynamic interactions between particles.

The interplay between the flow and moving phase boundaries produces diverse nonlinear effects in the two systems under discussion. For viscous drops, there occurs a hysteretic response of the drop shape to quasistatic change of the external flow vorticity, and we also observe period-doubling bifurcations leading to chaos, for periodically varying vorticity [4]. In the other system the interaction between regular particle arrays and Poiseuille flow results in propagation of particle displacement waves, sudden lattice rearrangements, order–disorder transitions, and fingering instabilities [5]. We elucidate the underlying physical mechanisms of these phenomena.

Our paper is organized as follows: In Section 2 we discuss the dynamics of viscous drops in external 2D linear flows with rotation. In Section 3 we analyze the collective dynamics of ordered particle arrays in Poiseuille flow in a parallel-wall channel. Our conclusions are presented in Section 4.

2. Hysteretic and chaotic drop dynamics

The evolution of a deformable viscous drop is considered in linear creeping flows with rotation. We focus on systems where the drop viscosity is much higher than the continuum phase viscosity. In the creeping-flow regime, the evolving boundary conditions due to the motion of the drop interface are the only source of nonlinear dynamics.

We find that nonlinear coupling of the drop deformation and rotation to the external flow results in drop bistability and hysteresis in quasistatic drop shape evolution. We also analyze a novel chaotic drop dynamics resulting from a period-doubling bifurcation cascade.

2.1. Viscous drop in creeping flows

We consider a viscous drop immersed in an incompressible fluid of a constant viscosity μ. The viscosity of the drop is $\hat{\mu} = \lambda\mu$

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doi:10.1016/j.physd.2009.11.013

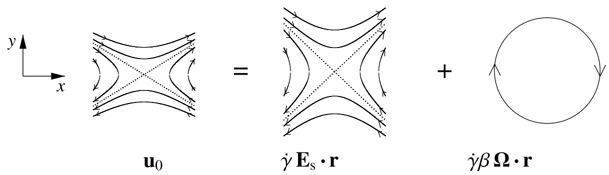

Fig. 1. Decomposition of a linear incident flow into pure strain and rigid-body rotation.

(where λ is the viscosity ratio), and the interfacial tension between the two phases is σ. The fluid velocity $\mathbf{u}$ and pressure p in the regions inside and outside the drop are described by the Stokes equations

$$\mu_i \nabla^2 \mathbf{u} = \nabla p, \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

where $\mu_i = \hat{\mu}$ or μ is the corresponding fluid viscosity. The nonlinear boundary condition on the drop interface is the balance of normal stress with the capillary pressure

$$[\hat{\mathbf{n}} \cdot \boldsymbol{\tau} \cdot \hat{\mathbf{n}}] = 2\kappa\sigma, \tag{3}$$

where $\boldsymbol{\tau}$ is the viscous stress tensor, $\hat{\mathbf{n}}$ is the outward normal unit vector, and κ is the local curvature of the interface.

The drop is subject to a 2D linear incident flow

$$\mathbf{u}_0(\mathbf{r}) = \dot{\gamma}(\mathbf{E}_s + \beta\boldsymbol{\Omega}) \cdot \mathbf{r}, \tag{4}$$

where $\dot{\gamma}$ is the strain rate, β is the dimensionless vorticity parameter, $\mathbf{r}$ is the position, and

$$\mathbf{E}_s = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \boldsymbol{\Omega} = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5}$$

are the symmetric and antisymmetric parts of the velocity gradient tensor. The symmetric part describes a purely straining flow, and the antisymmetric part corresponds to rigid-body rotation with the angular velocity $\omega = \frac{1}{2}\beta\dot{\gamma}$. The decomposition of incident flow (4) into the straining and vorticity components associated with tensors $\mathbf{E}_s$ and $\boldsymbol{\Omega}$ is sketched in Fig. 1.

Three dimensionless parameters characterize the dynamics of the viscous drop. The viscosity ratio λ describes the relative magnitude of dissipative forces in the drop phase and continuous phase fluids. The capillary number $\mathrm{Ca} = a\mu\dot{\gamma}/\sigma$ (where a is the radius of an undeformed drop) gives the ratio between the deforming viscous forces produced by the imposed flow (4) and the capillary forces driving the drop towards the equilibrium spherical shape. Finally, the vorticity parameter β describes the magnitude of the rotational component of the external flow relative to the extensional component.

2.2. Bistable stationary states and hysteresis

For sufficiently large viscosity ratios ($\lambda > 100$) and moderate capillary numbers (below the critical value for drop-breakup instability), two stable stationary drop shapes are found for a range of β between critical values β_1 and β_2. These two stationary states are illustrated in Fig. 2. The drop shape shown in Fig. 2(a) is elongated and nearly aligned with the extensional axis $x = y$; the shape shown in Fig. 2(b) is nearly spherical [6].

The elongated stationary shape results from the balance between drop deformation by the extensional flow component and drop relaxation due to the capillary forces. The respective time scales for the drop deformation and relaxation are $t_\gamma = \lambda\dot{\gamma}^{-1}$ and $t_\sigma = \lambda\mu a\sigma^{-1}$, both of which are proportional to the viscosity ratio

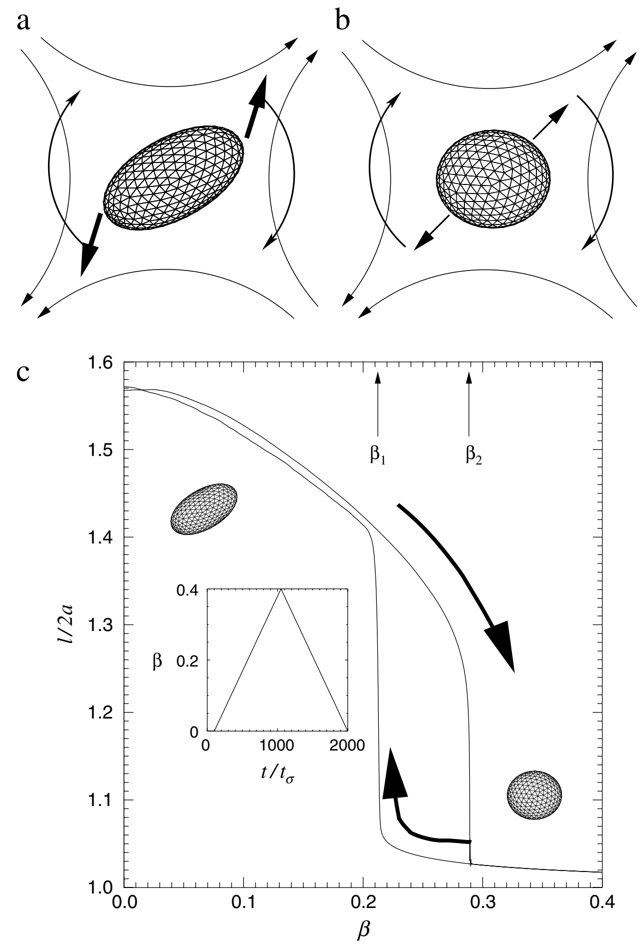

Fig. 2. (a) Surface-tension-stabilized elongated drop. (b) Rotationally stabilized compact drop. (c) Hysteresis of a highly viscous drop in 2D linear flow with varying vorticity. Results are from boundary-integral simulations with $\lambda = 200$ and $\mathrm{Ca} = 0.20$. Inset shows the dependence of vorticity on time.

for $\lambda \gg 1$. The drop deformation $D = (l - 2a)/a$ (where l denotes the drop length) is determined by the time scale ratio

$$D \sim t_\sigma/t_\gamma = \mathrm{Ca}. \tag{6}$$

Therefore, D is independent of the viscosity ratio in the limit $\lambda \to \infty$. The $O(\lambda^{-1})$ internal circulation inside an elongated high viscosity drop is weak for large λ. Thus the drop behaves like a rigid body whose equilibrium orientation results from the balance of the torques produced by the straining and rotational components of the external flow, as depicted in Fig. 2(a).

The compact stationary shape is stabilized by the circulation of the fluid inside the drop, which rotates with an angular velocity ω_d, nearly equal to the angular velocity ω of the external flow. Within each period of rotation the drop undergoes a small deformation produced by the straining component of the external flow, as schematically illustrated in Fig. 2(b). However, the deformation does not grow, because it is constantly convected away by the rotational component of the flow. Since the rotation occurs on the time scale $t_{\mathrm{rot}} = (\beta\dot{\gamma})^{-1}$, and the drop deforms on the much longer timescale $t_\gamma = \lambda\dot{\gamma}^{-1}$, we find that the drop deformation in the compact state,

$$D \sim t_{\mathrm{rot}}/t_\gamma = (\beta\lambda)^{-1}, \tag{7}$$

is small for $\lambda \gg 1$.

The existence of two stationary states implies a hysteretic drop response to quasistatic variation of vorticity β. Such hysteresis in