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Ultra-short scalar and vector solitons in self-inductively transparent media

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ABSTRACT

The analytical form of the one-soliton solutions to the Maxwell–Bloch equations is found without the slowly-varying envelope approximation with application to the ultra-short (few-cycle or sub-cycle) light pulses propagating in media of two-level atoms as well as to fluxons in the long Josephson junctions. Also, we discuss the dynamics of the ultra-short vector solitons propagating in specific three-level media and magnetic-flux transmission lines (of two long Josephson junctions sharing a common superconducting plate). Studies of the (ultra-short) pulse collisions lead to the prediction of pulse stability against the collisions. In particular, the collisions of the ultra-short vector solitons are investigated in detail. Their collision-induced polarization transform is found to be similar to the polarization transform of the vector (Manakov) solitons propagating in self-focusing media.

1. Introduction

The propagation of ultra-short (few-cycle and sub-cycle) solitons in nonlinear optical fibers has been studied theoretically for a long time and tested since the beginning of the microstructured-fiber production [1,2]. It is described with the modified (via adding a nonlinear differential term) nonlinear Schrodinger equation whose soliton solutions are well known [3,4]. Including the pulse polarization, the modified vector nonlinear Schrodinger equation takes the form

$$\mathrm{i}\frac{\partial \mathcal{E}_j}{\partial x} + \frac{\partial^2 \mathcal{E}_j}{\partial t^2} + \mu\left(1 + \frac{\mathrm{i}}{\omega}\frac{\partial}{\partial t}\right)\left[\left(\sum_{l=1}^{2} |\mathcal{E}_l|^2\right)\mathcal{E}_j\right] = 0 \quad (j = 1, 2), \tag{1}$$

where $\mathcal{E}_j$ denote the envelopes of the electric field components. The one-hump (ultra-short) pulses governed by (1) were not proved to be stable against the collisions (contrary to the solutions of its scalar counterpart) and the strict two-pulse solution of (1) has not been found [5]. However, due to the expectation of the long-time coherence of the intense (ultra-short) pulses, one is interested in the possibility of controlling the pulse polarization via the collisions with other pulses. In particular, it is of importance for potential applications of the vector solitons in collision-based information-processing schemes (performing logical-gate operations via the soliton collisions) [6–8].

Recent development in methods of the few-cycle and sub-cycle optical pulse generation, [9], has also led to the interest in the ultra-short solitons of self-induced transparency (SIT), (the electromagnetic pulses in homogeneous and unexcited media of two-level atoms interacting with light via dipole coupling [10]). The SIT effect was initially predicted by applying the slowly-varying envelope (SVE) approximation to the system of coupled Maxwell and Bloch equations of motion while the relevance of this approximation (especially for very-short pulses) has been contested since [11]. The major research effort in the direction of SIT for ultra-short pulses has been devoted to numerical studies of soliton stability in the few-cycle and sub-cycle regimes, and the inapplicability of the SVE approximation to solving the Maxwell–Bloch equations in these regimes has been confirmed [12,13]. Analytical studies of the few-cycle SIT (reviewed in [14]) are

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restricted to specific parameter regimes that enable reduction of the secondary (containing second differentials) Maxwell equation for the electric field to a first-order differential equation [15].

In the present report, I study the Maxwell–Bloch system formulating the model with the primary (containing first differentials) Maxwell equations. Changing the ratio of the electric and magnetic field amplitudes with relevance to that of the free electromagnetic wave and of the previously studied SIT pulses, we find the analytic form of the solitary-wave solution to the Maxwell–Bloch equations relevant to the ultra-short pulse regime without limitations to the parameter ranges. I show this wave to possess the property of the soliton to be stable against the collisions. The result is extended to the ultra-short vector pulses propagating in media of three-level atoms or atom-like quantum objects [16], which should be especially emphasized with connection to the vector-soliton applications mentioned in the first paragraph. In this context, let us also mention on recent interest in the collision-induced pulse-polarization switching in media of Λ-type tree-level configurations (the regime of the electromagnetically-induced transparency) due to different possibilities of the coherent preparation of stable states of the medium [17]. We analyze vector solitons (fluxons) propagating in a long double Josephson junction (built of two long tunnel junctions that share a common superconducting plate) which is equivalent to a medium of V-type or Λ-type atoms (of a twofold degenerated energy level) [18,19]. The descriptions of the pulse propagations in both the systems (atomic medium and long Josephson junction) are known to be similar (based on the Maxwell–Bloch equations) [20]. However, we avoid the widely used reduction of the relevant Maxwell–Bloch systems to the sine-Gordon equation (describing the scalar-pulse dynamics [20]) or to the coupled sine-Gordon equations (the vector-pulse [18,21]) which is an oversimplification with connection to the ultra-short pulse regime.

Let us mention that there are known predictions of specific few-cycle solitary waves that propagate in media of atoms with a permanent dipole moment (the so called Stark media) [14,22]. The present study deals with simple systems (media of atoms without any permanent dipole moment). It is restricted to the bright-soliton problem while different kinds of the scalar and vector pulses relevant to nonvanishing boundary conditions are behind of its scope [23,24].

The content of the following text is divided into the sections presenting: the dynamical model of the scalar and vector pulses of the SIT that is applicable to the atomic system as well as to the fluxon transmission lines (Section 2), its one-soliton solution found with use of the direct (Hirota) method (Section 3), the study of the pulse collisions using the inverse-scattering transform (Section 4), summary of the results (Section 5).

2. Model

We consider the medium of identical two-level atoms (whose energy eigenstates are denoted by $|\mathcal{D}\rangle$, $|\mathcal{C}\rangle$ and relate to the eigenvalues $\omega_{\mathcal{D}}, \omega_{\mathcal{C}}$, respectively) interacting with a light pulse via the dipole coupling. The state vector of the medium takes the form

$$|\Psi(x,t)\rangle = \mathcal{D}(x,t)\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{D}}t}|\mathcal{D}\rangle + \mathcal{C}(x,t)\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{C}}t}|\mathcal{C}\rangle \tag{2}$$

and its evolution is governed by the equations of motion

$$\mathrm{i}\frac{\partial(\mathcal{C}\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{C}}t})}{\partial t} = \omega_{\mathcal{C}}\mathcal{C}\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{C}}t} + 8\pi \mathrm{d}E\mathcal{D}\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{D}}t},$$
$$\mathrm{i}\frac{\partial(\mathcal{D}\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{D}}t})}{\partial t} = \omega_{\mathcal{D}}\mathcal{D}\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{D}}t} + 8\pi \mathrm{d}E^*\mathcal{C}\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{C}}t}, \tag{3}$$

where d denotes a dipole moment, $E(x,t) \equiv E_y(x,t) + \mathrm{i}E_z(x,t)$ — the electric component of the photon field. In order to simplify the dynamical (Maxwell) equation for the electric field, we generate the new dynamical (Bloch) variables

$$P \equiv \mathrm{i}2(\mathcal{C}\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{C}}t})(\mathcal{D}^*\mathrm{e}^{\mathrm{i}\omega_{\mathcal{D}}t})\mathrm{e}^{-\mathrm{i}\delta t}, \qquad D \equiv \mathcal{C}^*\mathcal{C} - \mathcal{D}^*\mathcal{D}, \tag{4}$$

where $\delta \equiv \omega_{\mathcal{D}} - \omega_{\mathcal{C}} + \omega$ denotes the optical detuning for $|\omega|$ denoting the main pulse (photon) frequency. The Maxwell equations

$$\frac{\partial H_z}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_y}{\partial x}, \qquad \frac{\partial E_y}{\partial t} + \frac{1}{\epsilon_0}\frac{\partial H_z}{\partial x} = -\frac{4\pi\rho}{\epsilon_0}\frac{\partial P_y}{\partial t}, \tag{5}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0}\frac{\partial E_z}{\partial x}, \qquad \frac{\partial E_z}{\partial t} - \frac{1}{\epsilon_0}\frac{\partial H_y}{\partial x} = -\frac{4\pi\rho}{\epsilon_0}\frac{\partial P_z}{\partial t}, \tag{6}$$

(here $\mathrm{i}dP\mathrm{e}^{\mathrm{i}\omega t} = P_y + \mathrm{i}P_z$ corresponds to the transition dipole moment of the two-level atom, ρ denotes the density of the atoms) simplify via the assumptions $H_z = \nu(\epsilon_0/\mu_0)^{1/2}E_y$, $H_y = -\nu(\epsilon_0/\mu_0)^{1/2}E_z$. Including the possibility of $\nu \neq 1$ is a softening of the condition ($\nu = 1$) used in previous approaches to the SIT, while $\nu = \text{const}$ enables the pulse dynamics to be determined by a single envelope function (similar to previously studied ones). The parameter ν will be shown to relate to the pulse width and velocity and it is stated below that there is no reason for its value to be equal to one. We introduce the envelope functions for the electric field and polarization following $E(x,t) = \mathcal{E}(x,t)\mathrm{e}^{-\mathrm{i}\omega[t-x/(\nu c)]}$, $P(x,t) = \tilde{P}(x,t)\mathrm{e}^{-\mathrm{i}\omega[t-x/(\nu c)]}$, $(c = 1/\sqrt{\mu_0\epsilon_0})$, rescale the field envelope $\tilde{\mathcal{E}}(x,t) = 8\pi d\mathcal{E}(x,t)$, and transform the space–time variables following $\tilde{x} = x$, $\tilde{t} = t - x/(\nu c)$. Then the system of the Maxwell–Bloch equations can be written in the reduced form

$$\frac{\partial E}{\partial \tilde{t}} = \frac{\nu c}{1-\nu^2}\frac{\partial E}{\partial \tilde{x}}, \qquad \frac{\partial \tilde{\mathcal{E}}}{\partial \tilde{x}} = -\mu\left(1 + \frac{\mathrm{i}}{\omega}\frac{\partial}{\partial \tilde{t}}\right)\tilde{P},$$
$$\frac{\partial \tilde{P}}{\partial \tilde{t}} = -2\tilde{\mathcal{E}}D, \qquad \frac{\partial D}{\partial \tilde{t}} = \tilde{\mathcal{E}}\tilde{P}^* + \tilde{\mathcal{E}}^*\tilde{P}. \tag{7}$$

Here $\mu \equiv 2(4\pi d)^2\rho\omega/(\epsilon_0\nu c)$. Unlike for the usual (many-cycle) pulses of the SIT, we cannot neglect the differential term on the right-hand side of the second of Eqs. (7) which is the source of difficulty for solving the system. The first of Eqs. (7) was neglected within