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Chaotic dynamics of resistively coupled DC-driven distinct Josephson junctions and the effects of circuit parameters

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ABSTRACT

Two distinct Josephson junctions (JJs) connected with a constant coupling resistance R_{cp} are theoretically considered to investigate the overall dynamics below and above the critical current I_c. The circuit model of the device is driven by two DC current sources, I_1 and I_2. Each junction is characterized by a nonlinear resistive–capacitive junction (NRCSJ). Having constructed the circuit model, time-dependent simulations are carried out for a variety of control parameter sets. Common techniques such as bifurcation diagrams, two-dimensional attractors and Lyapunov exponents are applied for the determination of chaotic as well as periodic dynamics of the superconducting junction devices. According to the findings, two states (namely superconducting and ordinary conducting) are determined as functions of the source currents. The chaotic current which flows through R_{cp} exhibits a very rich behavior depending on the source currents I_1 and I_2 and junction capacitances C_1 and C_2. The device characteristics are summarized by a number of three-dimensional phase diagrams in the parameter space. In addition, for certain parameters, hyper-chaotic cases with two positive Lyapunov exponents are encountered. In contrast to earlier studies claiming the need for a sinusoidal feeding current for generating a chaotic signal, our circuitry can easily generate one via a DC source.

1. Introduction

Approximately 40 years after the first observation of chaotic behavior in Josephson junction (JJ) devices, many researchers are still working on this area. This is because JJ devices have been used in many applications such as ultrahigh sensitive detectors and superconducting quantum interference devices (SQUIDs) [1,2]. The literature has shown that many simple nonlinear systems, including Josephson circuits, can exhibit chaotic behavior. In this manner, JJ devices could be useful for ultrahigh-speed chaotic generators for applications of code generation in spread-spectrum communications [3] and random key generation in secure communication and encryption [4–6]. In this respect, the dynamics of JJs is of great importance in contemporary superconducting electronics [7–9] so that junction devices can be used as high-frequency chaotic signal generators. On the other hand, JJs have become interesting tools for engineers and physicists in order to explore the basic concepts of chaotic dynamics and nonlinear media such as period-doubling, synchronization and bifurcation phenomena. For instance, Nayak and Kuriakose have investigated the effect of the phase difference of the applied field on the dynamics of mutually coupled JJs [10]. According to their findings, the dynamics of the system changes from chaotic to periodic motion for some cases of applied phase difference.

It can also be stated that the physical aspects and parametrical dependences of junction systems can be sensitively enlightened by both theoretical and experimental models. For example, the frequency of an external periodic signal has been used as a control parameter by Dana et al. [11] and two important routes to chaos, period doubling and torus breakdown have been observed in the case of different frequency ranges. According to Corato and co-workers, the Josephson current is controlled and modulated with high precision by adjusting a small transversal magnetic field parallel to the superconducting loop plane [12]. The authors also stated that the system could be used to realize a switchable flux transformer to couple quantum bits when desired as well as to vary the critical current of a flux qubit. From a theoretical point of view, there exist different models of JJs to clarify whether the junctions can be operated as a transmitter and receiver in chaotic digital communications. Shunted linear resistive–capacitive junction (RCSJ), shunted nonlinear resistive–capacitive junction (NRCSJ) and nonlinear resistive–capacitive–inductive junction (NRCLSJ) models can be seen in this regard [8,9,11,13–15]. While parallel resistance

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and capacitance are basically used in the case of RCSJs and NRCSJs, an additional nonlinear current–voltage (I–V) characteristic is determined in the latter case. In contrast to the earlier models, the NRCLSJ model includes a parallel inductance in addition to a resistance and a capacitance [9,11,14,15]. This model is frequently considered for high-frequency signal generation. The device models given above also indicate differences among the source currents. Basically, the first two models show chaotic behavior when the systems are driven by an external sinusoidal signal (AC), while the last one can generate chaos with a DC input signal. In some earlier studies, coupled and large-array junctions were considered especially for engineering purposes as well [13]. From an engineering point of view, a robust, sensitive and economical design of such high-frequency devices has a great importance since DC current feeding can be better than any oscillating external source in order to provide the above-mentioned factors. To our knowledge, coupled junctions are mostly driven by external sinusoidal signals. In this study, we focus on a more compact and economical model which is simply driven by DC source currents so that the negative impact of sinusoidal external source current can be eliminated.

The paper is organized as follows. Section 2 covers the DC-driven JJ model for a resistively coupled junction device and the corresponding formulation of the system equations. Section 3 includes using knowledge of the simulation process so that the dynamics of the system can be analyzed numerically. Some initial numerical details and the necessary conditions are also given in the same section. Section 4 introduces two different states (i.e. superconducting and normal) when the coupled JJ device is considered. Section 5 discusses the impact of non-identical junctions on coupling currents; meanwhile, some attractors and time series of voltage and phase differences are presented. The main results and discussions based on the device parameters for chaotic behavior are presented and confirmed in Section 6 by using three-dimensional (3D) phase and bifurcation diagrams as well as Lyapunov exponents. Finally, the concluding remarks are given in Section 7.

2. Circuit model of a superconducting device

In this section, we construct a model of two resistively coupled Josephson junctions (JJs), in which two non-identical nonlinear resistive–capacitive superconducting Josephson junction (NRCSJ) circuits are connected to form the circuit shown in Fig. 1. The circuit equations for each NRCSJ can be obtained using Kirchoff's law:

$$C_i\frac{\mathrm{d}V_i}{\mathrm{d}t}+\frac{V_i}{R_i(V_i)}+I_{c_i}\sin\theta_i=I_i-I_{cp},$$
$$\frac{\hbar}{2e}\frac{\mathrm{d}\theta_i}{\mathrm{d}t}=V_i, \tag{1}$$

where V_i is the voltage across the ith junction circuits ($i=1,2$), $\hbar=h/2\pi$ is Planck's constant, e is the electron charge, I_i is the DC current source, I_{c_i} is the critical junction current, and C_i represents the junction capacitance of the ith junction. The phase difference inside the JJ is denoted by θ. Note that the current I_{cp} through the coupling resistance R_{cp} is given as

$$I_{cp}=\frac{V_1+V_2}{R_{cp}}. \tag{2}$$

Generally, for any NRCSJ model [9,8,11], the nonlinear resistance $R(V)$ (i.e. voltage-dependent junction resistance) is defined by

$$R(V)=\begin{cases}R_n & \text{if } |V|>V_g\\ R_{sg} & \text{if } |V|\le V_g\end{cases}.$$

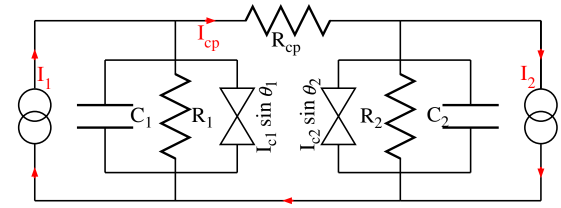

Fig. 1. Schematic representation of a resistively coupled DC-driven JJ system.

Here, $V_g=2\Delta/e$ is the gap voltage, R_n is the normal state resistance and R_{sg} is the sub-gap resistance of the JJ in the superconducting state.

The dimensionless form of the system equations from Eqs. (1) and (2) reads as

$$\frac{\mathrm{d}v_i}{\mathrm{d}\tau}+\frac{v_i}{r_i}+\omega_i^2\sin\theta_i=\omega_i^2\left(\frac{I_i-I_{cp}}{I_{c_i}}\right), \tag{3}$$

$$v_i=\frac{\mathrm{d}\theta_i}{\mathrm{d}\tau}, \tag{4}$$

$$I_{cp}=\frac{V_o}{R_{cp}}\left(v_1+v_2\right). \tag{5}$$

Here, we introduce the dimensionless normalized forms of the system parameters. The time, voltage and plasma frequencies are given by

$$\tau=\omega t,$$
$$V_o=\hbar\omega/2e, \tag{6}$$
$$\omega_i=\sqrt{2eI_{c_i}/(\hbar C_i)}/\omega,$$

respectively. In addition, the voltages on each JJ and the nonlinear resistances are represented by $v_i=V_i/V_o$ and $r_i=\omega C_iR_i(V_i)$. Note that ω is nothing other than a frequency normalization factor, and it is equal to $\omega=10^{12}$ Hz. Moreover, the quantity $1/r_i$ can be considered as a dimensionless damping factor.

In order to justify the effectiveness of all differential terms in Eq. (3), an analytical analysis can be carried out. Since the junctions include the nonlinear resistances, it is important to search the correct parameter range, which exhibits the overall nature of the complete system. This is because the system may yield to a fixed-point or periodic dynamics if the appropriate parameter set is not adjusted. For instance, Valkering et al. [16] explored the overdamped limit of a capacitively coupled JJ system and they numerically proved that a chaotic region exists for specific values of the β_c parameter (i.e. the multiplication of shunt resistance and coupling capacitance in their paper). Similarly, a brief analytical study can also give better understanding on the dynamics of the second-order differential equation in our case. In fact, the effectiveness of the terms in Eq. (3) can be determined for certain parameter regimes.

Initially, we should state that the coupling resistance R_{cp} plays an important role for the desired regime of the system (see also [10] for the importance of R_{cp}). In fact, the differential terms (i.e. $\frac{\mathrm{d}\theta_i}{\mathrm{d}\tau}$ and $\frac{\mathrm{d}^2\theta_i}{\mathrm{d}\tau^2}$ in Eq. (3)) cannot be negligible for the specified parameter range. Now, we wish to justify this point and prove that the nonlinear resistance $r_1(v_1)$ cannot always drive the system alone, but R_{cp} also contributes to the dynamics of the system. Let us assume that we are looking for the features of the JJ circuit at the left-hand side in Fig. 1. In this case, Eqs. (3)–(5) can be specified as

$$\frac{\mathrm{d}^2\theta_1}{\mathrm{d}\tau^2}+\alpha\frac{\mathrm{d}\theta_1}{\mathrm{d}\tau}+\frac{{\omega_1}^2V_0}{I_{c_1}R_{cp}}\frac{\mathrm{d}\theta_2}{\mathrm{d}\tau}+{\omega_1}^2\sin\theta_1=\frac{{\omega_1}^2I_1}{I_{c_i}}, \tag{7}$$