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Approaches to forecasting volatility: Models and their performances for emerging equity markets

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Abstract

In this study, the performance of a number of well-known statistical and stochastic models is analyzed when applied to forecasting returns and volatility in some financial markets. A new affine jump-diffusion model is also introduced and it is showed that this model achieves better results than existing ones when used to forecast volatility. The bases for developing the new model are some results showing that jumps introduced both in the return and volatility process play an important role in forecasting volatility, particularly in highly volatile markets, such as emerging equity markets.

1. Introduction

Many practical problems in modern finance require an understanding of the volatility and correlations of asset returns. It is well known that volatility estimation is important for investors taking on risk in order to generate higher expected returns. Volatility plays an important role in determining the overall risk of a portfolio and identifying hedging strategies that make the portfolio neutral with respect to market moves; moreover, volatility forecasting is also crucial in derivatives trading.

Unfortunately, traditional models turn out to be insufficient for capturing well-known volatility properties found empirically (cf. [17] and references therein), such as time varying, clustering, mean reversion, non-normality of returns, asymmetric effect of positive and negative shocks in returns, and volatility smiles.

Although the daily practice of finance uses volatility and correlation of asset returns as inputs, these quantities cannot be directly observed and must be estimated generally from historical observations on financial asset returns. Since different methods can give different covariance matrices, the choice of estimation procedure and therefore the asset returns model becomes very important.

It is well known that there are different approaches to forecasting volatility for option pricing. These approaches can be classified into two general classes: the one is based on statistical theory and the other is based on the implied volatility of market option prices.

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The standard historical volatility estimate is built on the assumption that the asset returns follow a constant multivariate normal distribution. However, empirical evidence proves that this approach does not explain the fact that distributions of the asset returns are fat-tailed. Also it does not explain the fact that volatility is time varying and it is a mean reverting and persistent process. Another related problem with this method is that it uses the entire history and it equally weights each day's return, no matter when the observation occurred.

To allow the volatility to be time varying, a common practice is to construct an exponential smoothing estimator using a fixed rolling window and a decay factor.

Extensive literature focuses on another estimator based on the ARCH–GARCH-type model that can be considered as a generalization of the rolling historical volatility and exponential smoothing. The ARCH model was introduced by Engle [15] as a model for macro-economic uncertainty and it was extended by Bollerslev [6] into GARCH. When used for volatility forecasting, the latter model has the advantage of being quite simple and it is able to capture the time-varying volatility, but it has two major shortcomings. First, it can be computationally intensive, second it represents shocks on returns as symmetric, whereas there is evidence for an asymmetric response for some markets, for example, the stock market. To deal with this problem, Nelson [30] proposed Exponential GARCH or EGARCH, which models the log of the variance, thus allowing for an asymmetric reaction to positive and negative shocks. The ARCH–GARCH-type model is used to analyze the variance movements, rather than being applied to forecasting. A problem in implementing these types of models is their estimation because they usually require a large number of observations. Another problem with the ARCH–GARCH models is that they focus on variance one step ahead so that they are not designed to forecast volatility for a long horizon (see also [8]).

Two other types of models are used in the literature to forecast volatility: SVOL (stochastic volatility) and JD (jump diffusion), which define returns through a stochastic model and a jump process, respectively. In the SVOL model, returns and volatility are modeled by two independent stochastic processes; this model describes the volatility as a time varying and a mean reverting processes. One of the disadvantages of this model is that, against the empirical evidence, positive and negative shocks have the same effects on volatility. The JD model defines returns as a jump process where the jump is expressed as a sum of normal variables and the number of variables is drawn from a Poisson distribution. If we allow the returns to be driven by a jump, the jump component is a potential source of market moves, positive and negative shocks will have asymmetric effect on the volatility. Still, the JD model is not able to explain the time-varying and mean-reverting properties of volatility.

Though these models are able to capture some aspects of financial markets and the main properties of their volatility behaviour (see [29,23,9,16,10]) they are not commonly used because they are quite hard to implement. In the literature, many authors have studied the combination of the last two models (see [4,2]) and they proved that, even if it is possible to cover very different aspects of market risk, these models do not sufficiently explain the substantial degree of "volatility of volatility" or the volatility smile. Moreover it is well known that the calibration of SVOL models is not easy when using the classical methodology of maximizing the likelihood function, since it is difficult to find an analytical expression for it.

Recently, Duffie et al. [13] proposed an affine jump-diffusion stochastic model SVOL-JJ with jumps in return and in volatility (on affine stochastic models and for their systematic theoretical approach and applications in finance see also [11,14,5]).

Pezzo and Uberti [31] developed some original studies on the forecasting capability of the SVOL model and the SVOL-JJ model of Duffie et al. [13] of the volatility of assets in very volatile financial markets. They showed how jumps in return and volatility in the affine jump-diffusion SVOL-JJ model play an important role in forecasting the volatility behaviour. These results led them to analyze affine jump-volatility models with different assumptions on the distributions involved and then to propose [31,33] a new bi-dimensional stochastic model, the SVOL-JJ-IG. In this model, shocks in returns and volatility are driven by two independent uncorrelated jump processes. Jumps in volatility are described by an Inverted Gamma distribution and jumps in returns by a Normal distribution. In this scenario, the probability distribution functions can be computed in a closed form. While in the literature, similar models are usually applied to simulate the underlying values used in option pricing, the proposed model is implemented and used to forecast the volatility of assets in very volatile markets, such as emerging equity markets.

The aim of this paper is to compare the effectiveness of the previous statistical and stochastic models in forecasting volatility, particularly in very volatile markets. In particular, we show how stochastic models involving jumps can be profitably used in volatility forecasting.

The paper is organized as follows. The predictive ability of statistical and stochastic models is compared in detail in Section 2; then, in Section 3, the new stochastic model with jumps, SVOL-JJ-IG, is outlined; in Section 4, the data and methodology used in the implementation of some models are described; the relative performance of the stochastic models and the results are gathered in Section 5; Section 6 presents the conclusions. Some theoretical details on stochastic models involved in this paper are collected in Appendix A.