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Oligopoly games with nonlinear demand and cost functions: Two boundedly rational adjustment processes

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Abstract

We consider a Cournot oligopoly game, where firms produce an homogenous good and the demand and cost functions are nonlinear. These features make the classical best reply solution difficult to be obtained, even if players have full information about their environment. We propose two different kinds of repeated games based on a lower degree of rationality of the firms, on a reduced information set and reduced computational capabilities. The first adjustment mechanism is called "Local Monopolistic Approximation" (LMA). First firms get the correct local estimate of the demand function and then they use such estimate in a linear approximation of the demand function where the effects of the competitors' outputs are ignored. On the basis of this subjective demand function they solve their profit maximization problem. By using the second adjustment process, that belongs to a class of adaptive mechanisms known in the literature as "Gradient Dynamics" (GD), firms do not solve any optimization problem, but they adjust their production in the direction indicated by their (correct) estimate of the marginal profit. Both these repeated games may converge to a Cournot–Nash equilibrium, i.e. to the equilibrium of the best reply dynamics. We compare the properties of the two different dynamical systems that describe the time evolution of the oligopoly games under the two adjustment mechanisms, and we analyze the conditions that lead to non-convergence and complex dynamic behaviors. The paper extends the results of other authors that consider similar adjustment processes assuming linear cost functions or linear demand functions.

1. Introduction

The classical oligopoly games, and the associated notion of Nash equilibrium, are based on quite demanding notion of rationality that includes assumptions on the available information set, on the firms' capability of extracting correct estimates from it and on the computational skills required to solve the optimization problems through which firms make their decisions. Many authors claim that such assumptions are too strong and that real producers are not so rational when they make their decisions. Moreover, as clearly stated by [3], the more refined the decision-making

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process, the more expensive it is likely to be. Therefore, especially when a (single) decision is not of crucial importance, no more than an approximate solution may be justified. They call "optimally imperfect decisions" the decisions such that "The calculation of the appropriate decision is simple, inexpensive, and well suited for frequent repetition". This point of view is also shared by other authors, see [30,11,12].

In this paper we consider an industry where n firms, indexed by $i = 1,\dots,n$, produce an homogeneous good with production levels q_i, $i = 1,\dots,n$, respectively. Strategic interactions arise because the price of the good depends on the total output of the industry according to a given inverse demand function

$$p = f(Q), \tag{1}$$

where $Q = \sum_{i=1}^{n} q_i$ is the total output of the oligopoly market. If $C_i(q_i)$ denotes the cost function of producer i, then his profit at time period t is

$$\pi_i(t) = p(t)q_i(t) - C_i(q_i(t)). \tag{2}$$

We assume that the market is characterized by the following nonlinear inverse demand function

$$p = a - b\sqrt{Q}, \tag{3}$$

which has been used in other oligopoly models and in the experimental economics dealing with learning and expectations formation (see e.g. [26,2]). The cost functions are also nonlinear, that is

$$C_i = c_{i0} + c_{i1}q_i + c_{i2}q_i^2, \quad c_{ik} \geqslant 0, \quad k = 0, 1, 2. \tag{4}$$

Indeed, quadratic cost functions are often met in applications. For example, in the modeling of renewable resources exploitation, such as fisheries, cost functions without the linear term (i.e. functions of the form (4) with $c_{i2} > 0$ and $c_{i1} = 0$) are usually considered.[1] Negishi [25] considers cost functions of the form (4) and investigates the role of convexity, i.e. the role of the coefficient of the term of second degree.

With these demand and cost functions the profit of firm i becomes

$$\pi_i(t) = q_i(t)(a - b\sqrt{Q}) - (c_{i0} + c_{i1}q_i(t) + c_{i2}q_i^2(t)). \tag{5}$$

In a classical Cournot oligopoly game, producers are assumed to be price takers and, at each time t, they decide their production levels by maximizing the expected profit

$$q_i(t+1) = \arg\max_{q_i} \pi_i^e(t+1) = \arg\max_{q_i}[p^e(t+1)q_i - C_i(q_i)], \tag{6}$$

where the expected price $p^e(t+1) = f(q_i, q_{-i}^e(t+1))$ and $q_{-i}^e(t+1)$ represents the output decisions of the other players as expected by player i. Moreover, the inverse demand function is assumed to be known by all firms. Cournot [14] assumes *naive expectations*, i.e. $q_{-i}^e(t+1) = q_{-i}(t)$, that is, each firm expects that the production of the other firms will remain the same as in the current period.[2] This implies that the condition for solving the optimization problem (6) gives implicit relations between productions at time t and those at time $t+1$. In the simplest (and lucky) case one can uniquely express $q_i(t+1)$ as functions of $q_{-i}(t)$

$$q_i(t+1) = r_i(q_{-i}(t+1)), \tag{7}$$

where r_i are called *reaction functions*, and (7) gives rise to a discrete-time dynamical system (*Best Reply Dynamics*). Cournot [14] studied the properties of the best reply dynamics when both the demand and the cost functions are linear getting also linear reaction functions. Other examples of explicit computation of the best reply dynamics are given in the literature, see e.g. [27], where a nonlinear demand function is considered together with linear cost functions, [21,7], where nonlinear cost functions are considered together with a linear demand function. In all these cases, the Nash equilibria correspond to the fixed points of the map (7), i.e. they are located at the intersections of the reaction curves.

In our case, considering the demand function (3) and the cost functions (4), the first order conditions for the optimization problem (6) become

$$\frac{\partial \pi_i^e}{\partial q_i} = (a - b\sqrt{Q}) - \frac{bq_i}{2\sqrt{Q}} - c_{i1} - 2c_{i2}q_i = 0 \quad i = 1,\dots n.$$

[1] In fisheries the most frequently used cost function is given by $C(x) = \gamma\frac{x^2}{X}$ where x is the quantity of fish harvested (production) when a fish stock X is available. This cost function can be derived from a Cobb–Douglas-type "production function" with fishing effort (labor) and fish biomass (capital) as production inputs (see [13,32,10]).

[2] Other kinds of expectations mechanisms can be used, such as adaptive expectations, see e.g. [31,8].