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Dynamic oligopolies with market saturation

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Abstract

Dynamic oligopolies are examined with single-product firms and saturated market. Both linear and isoelastic price functions are considered. The local asymptotical stability of the equilibrium is analyzed first, and then computer simulation is used to illustrate the attraction basins in nonlinear models.

1. Introduction

Dynamic oligopolies have been studied by many researchers during the last decades. The early results on single-product models are summarized in [3] and their multi-product extensions with many model variants and applications are given in Okuguchi and Szidarovszky [4]. In recent years an increasing attention has been given to the dynamic extensions of different oligopoly models [5,6]. For continuous systems the introduction of continuously distributed time lags and the resulting bifurcation make the dynamics more complex [7,1], for discrete models the application of the critical curve method is a useful tool to analyze global stability [2].

In all previously analyzed models it was assumed that the market demand function remained always the same regardless how many items had been sold there in the past. In this paper we will introduce market saturation and will examine its effect on market prices. Discrete time scales will be considered with linear and isoelastic price functions. Local asymptotic stability of the equilibrium will be first examined and computer simulation will be used to illustrate the attraction basins in selected nonlinear models.

2. Linear price function

In this section linear price function is assumed.

2.1. The dynamic model

Consider an n-firm oligopoly with market saturation. If $x_k(t)$ is the output of firm k at time period t, then in the same period the saturation level becomes

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$$Q(t) = \alpha Q(t-1) + \sum_{k=1}^{n} x_k(t), \tag{1}$$

where $\alpha \in [0,1)$ is the saturation rate of the market. At the end of time period t, the total quantity of products used by consumers is the sum of the new sales and the still working proportion of earlier sales.

The market price is assumed to be $B - AQ$ (A and B are positive parameters), the cost function of firm k is $a_k x_k + b_k$. So at time period t, firm k expects the profit to be

$$\pi_k = x_k(B - A(x_k + s_k^E(t) + \alpha Q(t-1))) - (a_k x_k + b_k) \tag{2}$$

with production level x_k, where $s_k^E(t)$ denotes its expectation on the output of the rest of the industry.

The profit maximizing production level is either zero or positive.

If it is zero, then firm k's interest is to leave the business, so in the long run we may assume that for all firms, the profit maximizing output levels are always positive. This is the case with a positive stable equilibrium, when the initial output levels of the firms are sufficiently close to the equilibrium. In this case

$$B - 2Ax_k - As_k^E(t) - A\alpha Q(t-1) - a_k = 0$$

implying that the best response of firm k is given as

$$x_k(t) = -\frac{1}{2}s_k^E(t) - \frac{\alpha}{2}Q(t-1) + \frac{B - a_k}{2A}. \tag{3}$$

Assuming adaptive expectations, with some $0 < K_k \leqslant 1$,

$$s_k^E(t) = K_k \sum_{l \neq k} x_l(t-1) + (1 - K_k)s_k^E(t-1). \tag{4}$$

Here K_k is called the speed of adjustment of firm k. Combining Eqs. (1), (3) and (4) a $(2n+1)$-dimensional discrete system is obtained:

$$x_k(t) = -\frac{K_k}{2}\sum_{l \neq k} x_l(t-1) - \frac{1-K_k}{2}s_k^E(t-1) - \frac{\alpha}{2}Q(t-1) + \frac{B - a_k}{2A}, \tag{5}$$

$$s_k^E(t) = K_k \sum_{l \neq k} x_l(t-1) + (1 - K_k)s_k^E(t-1) \tag{6}$$

and

$$Q(t) = -\frac{1}{2}\sum_{k=1}^{n} K_k \sum_{l \neq k} x_l(t-1) - \frac{1}{2}\sum_{k=1}^{n}(1 - K_k)s_k^E(t-1) - \frac{n\alpha}{2}Q(t-1) + \alpha Q(t-1) + \frac{nB - \sum_{k=1}^{n} a_k}{2A},$$

which can be reformulated as

$$Q(t) = -\frac{1}{2}\sum_{k=1}^{n}\left(\sum_{l \neq k} K_l\right)x_k(t-1) - \frac{1}{2}\sum_{k=1}^{n}(1 - K_k)s_k^E(t-1) + \frac{(2-n)\alpha}{2}Q(t-1) + \frac{nB - \sum_{k=1}^{n} a_k}{2A}. \tag{7}$$

Simple calculation shows that at the steady state

$$\bar{x}_k = \frac{(1-\alpha)B + \sum_{l=1}^{n} a_l - a_k(n+1-\alpha)}{A(1+n-\alpha)}$$

and

$$\bar{Q} = \frac{nB - \sum_{l=1}^{n} a_l}{A(1+n-\alpha)}.$$

System (5)–(7) is linear with coefficient matrix

$$\boldsymbol{J} = \begin{pmatrix} -\frac{1}{2}\boldsymbol{K} & -\frac{1}{2}\boldsymbol{D} & -\frac{\alpha}{2}\boldsymbol{1} \\ \boldsymbol{K} & \boldsymbol{D} & \boldsymbol{0} \\ -\frac{1}{2}\boldsymbol{1}^{\mathrm{T}}\boldsymbol{K} & -\frac{1}{2}\boldsymbol{1}^{\mathrm{T}}\boldsymbol{D} & \frac{(2-n)\alpha}{2} \end{pmatrix} \tag{8}$$