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Image analysis for the destabilization process of the petal pattern in a liquid crystal light valve with rotational optical feedback

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ABSTRACT

The destabilization process from the static petal pattern to the fluctuating petal pattern in a liquid crystal light valve with optical rotational feedback is investigated experimentally. When a spatial rotation of $\pi/6$ is imposed on the feedback, the six-fold static petal pattern appears. At a threshold voltage of the bifurcation, rotationally propagating patches arise on the petals. With the increase of applied voltage, the motion of the patches becomes irregular and fluctuations of the pattern develop. By measuring the time correlation function for the spatial wave numbers along the azimuthal direction in polar coordinates, it is found that a half of the fundamental spatial mode relevant to the six-fold petal pattern plays an important role in the bifurcation process. The development of fluctuations is also analyzed by the Karhunen–Loéve decomposition in detail.

1. Introduction

In the field of nonlinear dynamics, self-organizing patterns in liquid crystal systems have attracted a lot of attention during recent years. In particular, owing to the richness of the patterns that appear, self-organization and spatio-temporal dynamics in a liquid crystal light valve (LCLV) with optical feedback have received considerable interest in the last two decades. Starting from the pioneering work of Akhmanov et al. [1–3], many kinds of self-organized patterns in the LCLV have been investigated, both theoretically and experimentally [4–10]. A number of topics related to the mechanism of pattern formation in the LCLV have been reviewed by several researchers [11–13], with the LCLV often described as a Kerr-like medium, thus fitting in the general theoretical framework of a Kerr medium with optical feedback [14,15].

In the LCLV, a nematic liquid crystal and a photoconductor are sandwiched by ITO-coated glass plates. For the reflective type of LCLV, a dielectric mirror is coated on the liquid crystal side of the photoconductor. The liquid crystal side (reading side) and the photoconductor side (writing side) are optically isolated by the dielectric mirror. Under no illumination on the photoconductor, the liquid crystal molecules are influenced by the anchoring force and remain parallel to the glass plates because most of the applied voltage drops across the photoconductor. On the other hand, when the photoconductor is illuminated, the molecules tilt in accordance with the intensity of the illumination and the applied voltage. Such a molecular reorientation is called a Fréedericksz transition [16], and it occurs above a threshold voltage that corresponds to the electric field needed to overcome the restoring torque due to the elastic coupling of the molecules.

When an optical feedback is imposed on the LCLV, the Fréedericksz transition becomes subcritical and the system becomes bistable, with different orientation states coexisting for the same values of the applied voltages [17,18]. The bistable property for the tilt angle of the liquid crystal director arises in several voltage ranges [19]. In such bistable ranges of the voltage, when nonlocal spatial effects are added in the optical feedback, such as rotation of the backward image or diffraction over a free propagation length, self-organized patterns appear [5,13]. The pattern that appears depends on the type of feedback and on the control parameters of the experiment, the main ones being the voltage applied to the LCLV, the intensity of the input beam, the initial orientation of the liquid crystal director with respect to the input beam linear polarization, the rotation angle in the feedback loop, and the free propagation length.

In the present paper, we focus our attention on the petal patterns observed under a pure-interferential optical feedback

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(the free propagation length is put equal to zero thanks to a self-imaging system), when spatial rotation is introduced in the feedback loop [2–4,8]. When the feedback rotation angle Δ is commensurate with 2π, a static pattern consisting of $N = \pi/\Delta$ petals appears at the onset of the bistable voltage ranges. We will not deal with the case of incommensurate rotation, for which the petal pattern rotates with an angular velocity proportional to the difference from the commensurate angle [2,20]. The formation mechanism of the static petal pattern was theoretically investigated by Adachihara and Faid [4]. By increasing the applied voltage V, the static petal pattern becomes unstable at a threshold voltage V_{th}, and the pattern starts to fluctuate both in space and time. The fluctuations of the petal pattern increase with increasing V. Ramazza et al. [8] analyzed the applied voltage dependence of the spatial structure and the activated mode experimentally and theoretically by measuring the spatial autocorrelation function in polar coordinates. However, very little quantitative data are available on the destabilization process of the static petal pattern and the development of fluctuations.

In the present paper, we will investigate the destabilization process of the static petal pattern by measuring the spatial power spectrum and the autocorrelation functions in time. By adopting a functional fitting for the autocorrelation function, we will estimate the decay rate of the fluctuations. Besides the normal type of correlation function, we will also measure the time correlation for the spatial modes relevant to the spatial structure of the petal [21]. Since the petal pattern appears under the strong effect of rotational feedback, it is expected that spatial modes relevant to the rotational angle play an important role in the bifurcation towards a fluctuating pattern. For the case of an N petal pattern, the relevant modes can be identified as a spatial mode with wave number N along the azimuthal direction and its harmonic modes. By analyzing the time correlation for such specific modes, we will identify the most relevant modes able to trigger the bifurcation towards a chaotic behavior.

In addition to the correlation analyses, we will analyze the fluctuations of the petal pattern by adopting the Karhunen–Loéve (KL) decomposition [22,23]. The KL decomposition, which is equivalent to the singular value decomposition and principal component analysis, expands a sequence of patterns into mutually uncorrelated sequences of patterns. The uncorrelated sequences of patterns are ordered with respect to their energy. This is a familiar technique in the field of image analysis. While the bases of decomposition in image analyses, such as Fourier decomposition, are fixed [e.g. sin( ) and cos( )] in general, the bases of KL decomposition are not fixed but calculated from the observed patterns.

By applying the KL decomposition on the fluctuating image sequence, we can identify a small set of dominant modes that contain most of the relevant information. Moreover, we evaluate the global entropy, which is a measure of complexity for the fluctuations. Recently, the KL decomposition has been applied to analyze the dynamics of self-organized structures [24–26]. For the pattern in the LCLV, Pastur et al. [27] have investigated the destabilization process of a stripe pattern in the LCLV system by using the KL decomposition.

The paper is organized as follows. In Section 2, we will briefly describe the structure of the LCLV and the experimental setup. In Section 3, we will define the power spectrum and the correlation functions and then introduce the Karhunen–Loéve decomposition. In Section 4, we will present the experimental results and discuss the characteristic features of the fluctuations. Finally, in the last section, we will summarize the results of the paper.

2. Experimental setup

The experimental system to observe the petal pattern under optical feedback is schematically illustrated in Fig. 1. The LCLV used in the experiment is a Hamamatsu PAL-SLM. A laser beam emitted

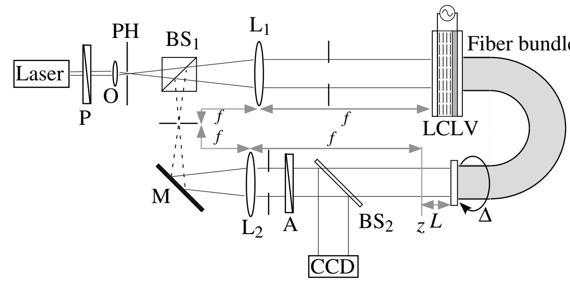

Fig. 1. Optical system to observe the petal patterns.

from a He–Ne laser (Neoark model 35) is enlarged and collimated by an objective O, a pinhole PH and a lens L_1 ($f = 250$ mm). The polarizer transmits the horizontal component of the beam. The diameter of the diaphragm located in front of the LCLV is 10 mm. In order to exploit the maximum optical anisotropy (birefringence) of the liquid crystal [28], the director angle is set $\pi/4$ from the direction of the polarizer P.

The laser beam injected into the nematics is reflected back by the dielectric mirror of the LCLV, and then is fed back to the writing side of the LCLV by the lenses L_1, L_2 ($f = 250$ mm), the beam-splitter BS_1 and the fiber bundle. Owing to the birefringence of the nematics, the outgoing light from the reading side of the LCLV is elliptically polarized. The phase shift between the ordinary and the extraordinary rays, ϕ, depends on the inclined angle of the nematics. As the analyzer A transmits only the vertical component of the elliptically polarized light, the intensity of the writing light I_w is written as [13]

$$I_w = R\left\{e^{-\gamma}\frac{I_0}{2}(1-\cos\phi)\right\}, \quad (1)$$

where I_0 represents the intensity of laser beam, γ is an effective attenuation coefficient for the optical feedback, and R is the rotational operator representing the fiber bundle rotation. In the present optical system, the analyzer converts the phase modulation introduced by the liquid crystal into an intensity modulation in the feedback loop. The free end of the fiber bundle, which is located at the focal plane of L_2, is mounted on a precision rotating stage. The feedback light is rotated by $\Delta = \pi/6$, which leads to the appearance of the six-fold petal pattern in several voltage ranges. To observe the feedback image, 5% of the feedback light is sampled by a beam splitter BS_2. A CCD camera and a frame grabber card (Scion Corporation LG-3) capture successive $T = 600$ frames, with a frame rate $\Delta t = 0.04$ s. The size of a single image is 512×512 pixels. The intensity of image at each pixel is digitized into 256 gray values. The observed images are analyzed by an image processing software (ImageJ) and originally developed programs. An example set of images of the six-fold petal pattern is displayed in Fig. 2.

The applied voltage V, supplied from a synthesizer (NF Corporation WF1944), was varied from 1.89 V to 2.25 V, which is the second bistable regime after the Fréedericksz transition. The frequency of the applied voltage, the rotation angle and the intensity of laser beam were fixed at 1 kHz, $\pi/6$ and 50 $\mu W/cm^2$, respectively. Five runs of the same experiments were performed at room temperature.

3. Analysis

3.1. Fourier analysis in polar coordinates

Since the static petal pattern has a radial symmetry, we have analyzed all the images in polar coordinates. Thus, the observed pattern in Cartesian coordinates $I(x, y, t)$ is converted into polar coordinates $I(r, \theta, t)$, and the space–time plot of the image is calculated at specific values of r.