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Generation of polarization singularities in the self-focusing of an elliptically polarized laser beam in an isotropic Kerr medium

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HIGHLIGHTS

- The possibility of polarization singularities emergence during self-focusing in Kerr medium is shown.
- In the case of initially axially symmetrical beam formed C-lines have the shape of circumferences.
- In the case of the beams with initially elliptical cross-section formed C-lines are three-dimensional closed curves.
- Total topological charge of C-points, nucleated during self-focusing, is zero in each cross-section of the beam.

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ABSTRACT

We have numerically and analytically shown that polarization singularities can emerge when a homogeneously elliptically polarized light beam undergoes self-focusing in an isotropic third-order Kerr medium without frequency and spatial dispersion (fused silica, liquids, gases etc.) In the case of axially symmetric beam the emerging C-lines have the shape of circumference with the center at the beam's axis and they are located in the separate transversal planes in the medium. If the axial symmetry of the incident beam is broken then the even number of C-points with opposite topological charges are nucleated in the medium. They exist in a certain propagation coordinate range and then they collide and annihilate each other.

1. Introduction

When an elliptically polarized laser beam has the power greater than the critical, it undergoes self-focusing [1], which is described by a nonlinear parabolic equations system for the slowly varying amplitudes $E_\pm$ of the circularly polarized components (partial beams) of the beam's electric field. The first attempts to describe the complex spatial dynamics of the intensity and polarization distributions of the propagating radiation were taken in [2–6]. In [5,6] the equations system for the dimensionless waist sizes $f_\pm$ of the circularly polarized partial beams was solved numerically in the paraxial approximation and the monotonic and non-monotonic regimes of $f_\pm$ behavior were found. Also, oscillation and other regimes of polarization ellipse's ellipticity degree dependence on the propagation coordinate were found in [7] by numerical simulations. The self-action of an elliptically polarized beam with the Gaussian profile in an isotropic medium with spatial dispersion of the cubic nonlinearity is thoroughly studied in [8,9]. The analysis of the equations for $E_\pm$ using the method of moments in aberration-free approximation established, that the intensity distribution of an initially homogeneously elliptically polarized beam loses its Gaussian shape during the propagation and its polarization distribution becomes inhomogeneous in its cross-section plane [8]. The obtained dependencies of the intensity, ellipticity degree and angle of orientation of the polarization ellipse's major axis on the spatial coordinates were shown to be in a reasonable accordance with the data of numerical simulations [9,10] near the beam's axis. Herewith, the observed transversal distributions of the intensity and polarization in the propagating beam differed greatly from those at the medium's border.

The complicated spatial dynamics of the intensity and polarization in the process of self-focusing lets one to expect the formation of the special points in the transversal distribution of the beam's electric field—the points of polarization singularity (C-points). In these points the elliptical polarization of the propagating radiation

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degenerates into a circular one, and one of the two circularly polarized components of the radiation has a phase dislocation [11,12]. These points are distinguished by their topological charge and type. The first quantity is the variation of orientation angle of the polarization ellipse's major axis calculated along a small closed contour surrounding the C-point and normalized on 2π. Usually, topological charge can be either 1/2 or $-1/2$ [11]. Besides that, C-points are distinguished by their topological type, each of which corresponds to the qualitatively different distribution of the polarization ellipses near the singularity. Three topological types are known in literature: lemon (the charge 1/2), monstar (the charge 1/2) and star (the charge $-1/2$) [11].

Light singularities (optical vortices and C-points) are of special interest in problems of nonlinear optics. The emergence of structurally stable singularities was experimentally observed in various nonlinear processes in crystals: photorefraction [13,14], Pockels effect [15–17], Faraday effect [18]. A number of theoretical research reveals the complicated dynamics of C-points generated in the bulk and on the surface of isotropic nonlocal media by the fundamental beams carrying polarization singularities in three-wave mixing processes [19,20]. The dynamics of optical singularities in processes of filamentation and supercontinuum generation were studied in [21–23].

Until now, the possibility of the formation of polarization singularities in Gaussian beam, self-focusing in nonlinear isotropic medium, was not investigated, except for [10], in which the singularities did non appear because of the specific range of the nonlinear medium's parameters considered. In the present paper we study the possibility of C-points formation during the self-focusing of an initially homogeneously elliptically polarized Gaussian beam, which propagates in an isotropic medium with cubic nonlinearity without spatial and frequency dispersion. The examples of such kind of media can be amorphous solids (fused silica [24,25]), liquids (CS_2 [26], water [27], organic liquids with strong cubic nonlinearity [28]), gases (air [29,30], argon [31]). Hence, the external group of symmetry of all material tensors of the medium is $D_{\infty h}$.

2. Model of self-focusing of arbitrary polarized beam

Let an initially homogeneously elliptically polarized monochromatic light beam fall normally along the z axis on the flat boundary $z = 0$ of an isotropic medium with cubic nonlinearity. We assume that the amplitudes $E_\pm$ of the circularly polarized components of its electric field vector are given by following expressions:

$$E_\pm(x, y, z = 0) = [I_0(1 \pm M_0)/2]^{1/2} \times \exp\left(-\frac{(x/b)^2 + y^2}{2a^2}\right)[1 + \xi(x, y)]. \quad (1)$$

Here I_0 is normalized intensity of the beam, M_0 is the ellipticity degree of the polarization ellipse at the medium's border and a is the beam's radius at e^{-1} level. The value $M_0 = 0$ corresponds to linearly polarized wave, $M_0 = 1$ ($M_0 = -1$) corresponds to the left-hand (right-hand) circularly polarized wave. The parameter $b \geq 1$ is the axes ratio of the transversal intensity profile, which has the shape of an ellipse with major axis parallel to x axis. The noise $\xi(x, y)$, the influence of which will be discussed in the final part of the paper, has the dispersion 0.03 and the correlation radius $0.1a$.

The equation system for the normalized amplitudes $A_\pm = E_\pm/I_0^{1/2}$ in dimensionless coordinates: $x' = x/a$, $y' = y/a$, $z' = z/(ka^2)$ (k is a wave number) has the following form [32]

$$2i\frac{\partial A_\pm}{\partial z} = \Delta_\perp A_\pm + R(|A_\pm|^2 + 2|A_\mp|^2)A_\pm. \quad (2)$$

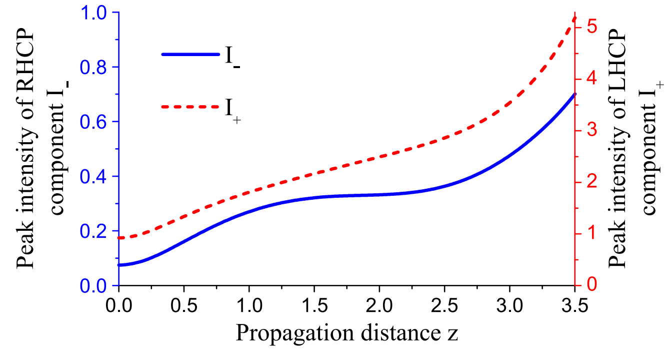

Fig. 1. Dependencies of peak intensities of the right-hand circularly polarized (RHCP) I_- and left-hand circularly polarized (LHCP) I_+ components of an axially symmetrical beam on the propagation coordinate z. The beam's polarization ellipse ellipticity degree $M_0 = 0.85$ and the nonlinearity parameter $R = 3.4$.

Here and further on we omit primes for brevity. In (2) $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transversal Laplacian, $R = n_2k^2a^2I_0/6n_0$ is proportional to the ratio of the beam's power and the critical power of self-focusing, n_0 and n_2 are the medium's linear and nonlinear refractive indices respectively. Linearly polarized axially symmetrical beam ($M_0 = 0$, $b = 1$) collapses if $R > 3.77/3$ [33], the equation for the critical power of self-focusing of a beam with elliptical intensity profile ($b \neq 1$) was obtained in [34]. The self-focusing can be described as the propagation of two partial beams with opposite circular polarization, each of which is subjected to the self-action as well as influenced by the other beam via nonlinear cross-action.

3. Discussion of the results

Our investigations show that the inequality

$$|A_-(x, y, z)|^2 \ll |A_+(x, y, z)|^2$$

is valid when $M_0 > 0.8$. Therefore, it is natural to expect the formation of C-lines with left-hand circular polarization in the bulk of the medium. These lines are determined by the equation $A_-(x, y, z) = 0$. In the case of axial symmetry ($b = 1$ and $\xi(x, y) = 0$) this expression is written like $A_-(r, z) = 0$, where $r = (x^2+y^2)^{1/2}$, and it is actually a system of two equations (for real and imaginary parts), each of which determines a family of curves in space (r, z). The curves of two families generally intersect with each other at separate values $\{r_m, z_m\}$ (where $m = 1, 2, 3 \ldots$), which correspond to the formation of separate C-lines in the bulk of the medium. Each C-line has the shape of a circumference with radius r_m that fully lies in the transversal plane $z = z_m$. The fact that C-lines are formed at separate distances z_m from the mediums border can also be shown by derivation of the implicit function $A_-(r, z) = 0$ with following substitution of $\partial A_-/\partial z$ from (2):

$$\frac{dr_m}{dz_m} = \frac{i}{2}\left[\frac{1}{r_m} + \frac{\partial^2 A_-}{\partial r^2}\left(\frac{\partial A_-}{\partial r}\right)^{-1}\right], \quad (3)$$

where the partial derivatives are taken at $\{r_m, z_m\}$. Here we also use the axially symmetric form of the transversal Laplacian $\Delta_\perp = \partial^2/\partial r^2 + r^{-1}\partial/\partial r$. Generally, the derivative (3) has non-zero imaginary part that indicates the absence of the C-lines in the planes adjacent to $z = z_m$. We confirm the formation of the C-lines by the numerical simulations, when the beam (1) undergoes self-focusing in Kerr medium, choosing the following parameters $M = 0.85$ and $R = 3.4$.

Fig. 1 shows the z-dependencies of normalized peak intensities $I_\pm(z) = \max_r\{|A_\pm(r, z)|^2\}$ of the circularly polarized components of electric field vector. Both functions grow monotonically when z