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Simple nonlinear models suggest variable star universality

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HIGHLIGHTS

- Simple nonlinear models connect golden stars and fractal spectra.
- Variable stars may reflect universal nonlinear phenomena common to simple systems.
- Simple ratios define some astrophysical resonances, golden ratios describe others?
- Facilitate a synergy between nonlinear dynamicists and astronomers.
- Suggest new directions for variable star astronomy.

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ABSTRACT

Dramatically improved data from observatories like the CoRoT and Kepler spacecraft have recently facilitated nonlinear time series analysis and phenomenological modeling of variable stars, including the search for strange (aka fractal) or chaotic dynamics. We recently argued [Lindner et al., Phys. Rev. Lett. 114 (2015) 054101] that the Kepler data includes "golden" stars, whose luminosities vary quasiperiodically with two frequencies nearly in the golden ratio, and whose secondary frequencies exhibit power-law scaling with exponent near −1.5, suggesting strange nonchaotic dynamics and singular spectra. Here we use a series of phenomenological models to make plausible the connection between golden stars and fractal spectra. We thereby suggest that at least some features of variable star dynamics reflect universal nonlinear phenomena common to even simple systems.

1. Introduction

The quality of the best stellar brightness time series has recently improved to the point where sophisticated nonlinear analysis becomes possible. For example, we recently used Kepler spacecraft data to analyze RR Lyrae variable stars [1,2] and identify a class of "golden" stars, whose luminosities vary quasiperiodically with two frequencies nearly in the golden ratio, and whose secondary-frequencies exhibit power-law scaling that suggests strange nonchaotic dynamics [3–5].

Many RR Lyrae and Cepheid multifrequency variable stars appear to cluster [6–10] about distinct frequency ratios, including a ratio of approximately 0.62, as in Fig. 1. A nonlinear dynamics perspective [11] immediately suggests that this is the (inverse) golden ratio $1/\varphi = 0.618\ldots$, a frequency ratio that features prominently in quasiperiodic systems and the famous KAM perturbation theorem [12].

Does the multifrequency variable star clustering near the golden ratio result from complicated and particular interactions of higher-order non-radial stellar modes [8]? Or does it reflect some universal behavior, like the critical exponents that characterize phase transitions, which are common to even simple nonlinear models? Are these stars *accidentally* golden or *fundamentally* golden?

This article presents a series of elementary nonlinear models or analogues of golden stars. These phenomenological models complement ongoing work to develop ab initio hydrodynamic variable star models [13,14]. Instead of a bottom-up derivation from variable star hydrodynamics to golden-ratio quasiperiodicity and fractal scaling, which may be very difficult, we offer a top-down approach consisting of a sequence of simple models

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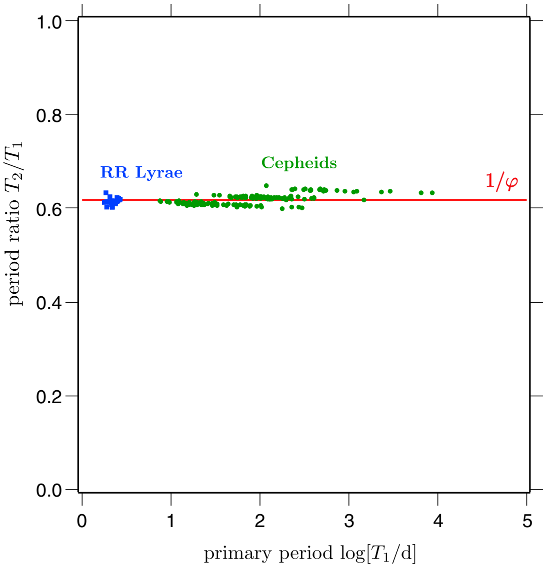

Fig. 1. Variable star Petersen diagram of period ratio T_2/T_1 versus primary period T_1/d in days has many multifrequency variable stars clustering near the inverse golden ratio $1/\varphi$. This is a rescaled version of Moskalik's Fig. 2 [6].

designed to make such a connection credible. The success in nonlinear dynamics of simple models embodying universal features may thereby generalize to variable star astronomy.

We first heuristically motivate the spectral distribution metric used to identify strange nonchaotic dynamics or singular spectra. After summarizing our most recent nonlinear golden star analysis and reviewing some of the relevant and unique properties of the golden ratio φ, we describe the models. Simple network models demonstrate how to easily or subtly introduce the golden ratio in stellar caricatures. Potential energy models create stylized versions of golden star spectral distributions or dynamical attractors. An unforced generalized Lorenz flow exhibits golden star scaling. A twist map suggests circumstances in which some dynamical systems can evolve into golden ratio configurations. Finally, we suggest possible new directions for astronomy from nonlinear dynamics.

2. Spectral distribution

Analysis of the Fourier spectrum of a time series can reveal subtle nonlinear dynamics [15]. In particular, strange nonchaotic dynamics [3–5] with singular spectra [16,17] is dynamics between order and chaos first identified numerically and analytically in quasiperiodically forced systems in the 1980s. The obvious strange nonchaotic signatures of negative maximum Lyapunov exponent and fractal geometry are often difficult to observe, especially in experimental data. However, scale-free power law scaling of a rich frequency spectrum provides a more practical signature, which we have recently discerned in golden stars [1]. Since this signature is not well known outside the nonlinear dynamics community, we here provide a heuristic derivation of it.

Consider a continuous signal $x[t]$ with two prominent incommensurate frequency components $f_1 = 1/T_1$ and $f_2 = 1/T_2$. Strobe the signal at the primary period T_1 and plot its values versus time modulo the secondary period T_2 in the Poincaré section

$$\{t_n, x_n\} = \{nT_1 (\mathrm{mod}\ \ T_2), x[nT_1]\}\,, \tag{1}$$

where n are integers. If a function $x_n = S[t_n]$ represents the section, is it smooth? Expand the function as a Fourier series

$$S[t] = \sum_{m=-\infty}^{\infty} \hat{S}_m e^{i2\pi mt/T_2}, \tag{2}$$

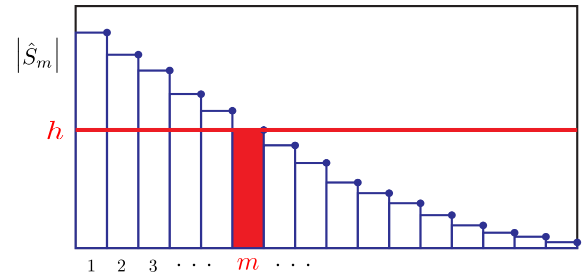

Fig. 2. In a schematic spectrum, m peaks (or bin heights) are higher than the mth peak.

with derivatives

$$\partial_t^k S[t] = \sum_{m=-\infty}^{\infty} \hat{S}_m (i2\pi m/T_2)^k e^{i2\pi mt/T_2}. \tag{3}$$

For smooth sections, expect the Fourier coefficients $\hat{S}_m$ to decay exponentially, so all the derivatives also decay (as the exponential overwhelms any power). For nonsmooth sections, expect the Fourier coefficients to decay slower, as a power law, so that some derivatives diverge. Specifically, for large Fourier modes m, expect

$$h \equiv \left|\hat{S}_m\right| \sim h_0 \begin{cases} e^{-m/b}, & \text{smooth,} \\ m^{-1/b}, & \text{nonsmooth,} \end{cases} \tag{4}$$

where $b > 0$. Invert to get

$$N \equiv m \sim \begin{cases} -b \log[h/h_0], & \text{smooth,} \\ (h/h_0)^{-b}\,, & \text{nonsmooth.} \end{cases} \tag{5}$$

Since an averaged spectrum decreases with mode number or frequency, so that m Fourier coefficients are higher than the mth coefficient, as in Fig. 2, reinterpret $N \equiv m$ to be the number of spectral peaks higher than the threshold height h. Numerical experiments indicate that this relation continues to hold for more general, irregular or sparse spectra.

Eq. (5) power-law spectral distribution $N \sim N_0 h^{-b}$ associated with nonsmooth Poincaré sections has been demonstrated in mechanical [18] and electrical [19] experiments and appears to characterize golden stars [1]. The experiments are typically optimized by driving quasiperiodically at two frequencies that are not only incommensurate, but are as incommensurate as possible, so that their ratio is golden (as discussed in Section 4). The mechanical experiment, for example, obtained good results and power-law scaling by driving near – but not exactly at – the golden ratio.

3. Stellar analysis

As an example of our stellar nonlinear analysis, consider the star of our previous paper, KIC 5520878, whose normalized brightness or flux F_N varies as in Fig. 3(a), where Kepler space telescope long cadence data has been conservatively detrended and standardized to zero mean and unit variance [1]. Although Kepler's time series are of unprecedented quality, they do contain both small and large gaps. Consequently, to estimate the frequency content of the time series, we do not use the Fast Fourier Transform algorithm, which assumes equally spaced points. Rather, we use Least Squares Spectral Analysis [20], effectively fitting sinusoids to the data, by computing the Lomb–Scargle periodogram $\hat{F}_N^2$ and its square root, as in Fig. 3(b).

The primary frequency f_1 and its overtones nf_1 create the repetitive non-sinusoidal waveform, while the secondary frequency $f_2 \approx \varphi f_1$, a golden ratio higher, modulate the maxima and minima. The spectrum is actually very rich (or very rough); it is not a discrete