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The hydrodynamics of flexible-body manoeuvres in swimming fish

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Abstract

Swimming in flexible-bodied animals like fish is characterised by a travelling wave passing along the spinal chord of the body. Symmetric transverse undulations of the body generate thrust and propel the fish forward. Turns are effected by generating an asymmetric transverse movement of the fish body, frequently as a C-shaped bend. Typical fish swimming speeds allow for simplifying assumptions of incompressible and inviscid flow. The objective of the current work is to use existing theoretical models developed for forward swimming, to analyse fish turns. Lighthill's classical elongated-body theory for fish swimming forms the fundamental basis for the 3D flow model and 'recoil' correction concept implemented here. In the methods developed here, transverse motion of a thin 'waving' plate is prescribed by a displacement signal acting along the midline, for finite time t_o. Lighthill's approach to calculate the rigid-body motion or 'recoil' correction is implemented to ensure zero net force and moments act on the body. Accordingly, angular and transverse motion are computed and final orientation of the plate after the manoeuvre is calculated. A 3D boundary-value algorithm has been developed using a vortex lattice method. The essential methodology, modifications for turning and comparisons with the analytical methods in the small and large aspect ratio limits are presented.

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1. Introduction

Fish are a typical example of flexible bodies swimming in an inviscid flow (Reynolds numbers $> 10^5$). Most fish swim in an aquatic environment that practically eliminates the effects of gravity. They have evolved different forms of swimming depending upon several factors, such as habitat and feeding habits. Gray's [1] pioneering experiments helped to characterise fish swimming in terms of a wave of muscular contraction passing down the length of the body. Symmetric transverse undulations of the body generate thrust and propel the fish forward. However, fish rarely swim in straight lines at constant speeds. More often, they tend to drift or swim slowly, occasionally indulging in rapid turns or a fast start in order to catch prey or escape from a predator. The objective of this paper is to examine the hydrodynamics of flexible bodied swimmers to understand the turning mechanics.

Lighthill [2] was the first to apply the methods of slender-body theory to an undulating body swimming in an inviscid fluid medium. Wu [3] modified the methods of thin airfoil theory to analyse the motion of a waving 2D plate. Both these methods allow for calculations of thrust, side force and yaw moment. Lighthill's theory is most likely to be applicable to long slender eel-like fish without prominent body-fins and a gradual taper in dimensions to the caudal fin. These theories have been further extended to account for body-taper at the caudal-peduncle and fin protrusions [4], time-varying forward speeds [5] and large amplitude motion [6]. Comparison of these methods is often done with numerical methods. Cheng et al. [7] first developed the vortex lattice method for rectangular, infinitely thin waving plates. Hill and Pedley [8,9] extended the method to examine large amplitude forward swimming. The full panel method for fish models of realistic thickness has been developed extensively by the Triantafyllou group at MIT [10, 11].

Significantly less attention, experimental or theoretical, has been devoted to studying turns however. Gray's investigations did include observations of turning fish [12]. Through

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time-sequence photographs, he was able to show the body-flexure involved in performing turns. In general however, most observations on manoeuvring fish have been on starts from rest [13]. Webb [14] recently examined turns in trout and bass to measure their turning radius and dependence on speed and acceleration. Theoretically, Weihs [15] applied the large amplitude version of Lighthill's theory [6] to model a turn, using Gray's experimental observations [12]. Although fins are important for turning in many species, they introduce complexities that Lighthill's theory of an undulating body does not account for. Recently Wolfgang et al. [10] qualitatively compared the flow field from PIV for a Giant Danio (*Danio Malabaricus*) performing a C-turn with their flow computations from an unsteady 3D numerical panel method. Other studies on manoeuvring swimmers have typically used methods based on linear control theory [16–18]. The key problem with large amplitude turns lies in the complex coupled interaction of the active bending body with the fluid, and in particular interaction with the vortex wake. Furthermore, the internal dynamics, both active muscle mechanics and passive visco-elastic deformations, depend on the inertial and hydrodynamic forces.

The essential 2D and 3D methods developed by Wu and Lighthill, respectively, have formed the basis from which the turning models were developed. Here we analyse how simple rectangular, thin, flexible-body swimmers manoeuvre. A numerical boundary value method using the 3D vortex lattice approach was developed to compare the validity and applicability of the theoretical methods, using a variation of the straight swimming techniques. The asymmetric manoeuvre is prescribed as a deflection about of the midline and the rigid-body motion or 'recoil correction' parameters are computed based on the inertial and hydrodynamic forces and moments. The straight swimming methods are summarised in Section 2 and the turning methods and corresponding results are presented in Section 3.

2. Forward swimming

This section examines the methodologies employed for analysing forward swimming in undulating bodies and compares some results from the analytical and numerical vortex methods.

2.1. Analytical swimming models

The linear swimming models assume the flexible-body swims at constant forward speed such that the thrust force balances fluid viscous resistance. Reynolds number is considered to be high enough ($\sim > 10^5$) for viscous effects to be confined to the boundary layer. Equations are solved in a body frame of reference with the fluid moving at velocity U_∞ along the x direction (from head to tail). For both analytical models, the fish is represented by an undulating midline with prescribed mass distribution. In this paper, Lighthill's 3D 'elongated-body' theory is examined briefly [2]. The 2D 'waving plate' method developed by Wu [3] will not be discussed here although results from this theory will be compared with those from the numerical methods.

Lighthill's elongated-body theory: This theory assumes transverse-body dimensions and deflections are an order of magnitude smaller than the body length and there exists a gradual variation in cross-section profile. The body has a stretched-straight configuration such that no resultant normal force acts at any point along the body. It executes transverse motion $h_z(x, t)$ along the perpendicular z direction.

Slender-body theory describes flow around the body to be a linear combination of the ambient steady flow and perturbations in the fluid flow induced by body deflections. The transverse velocity is given by,

$$v(x,t) = \frac{\partial h_z}{\partial t} + U_\infty \frac{\partial h_z}{\partial x} \tag{1}$$

relative to the free-stream. This imparts a momentum $\rho A(x) v(x, t)$ per unit length of the body, where $m(x) = \rho A(x)$ is the added mass per unit body length. In Lighthill's model $m(x)$ is approximately the mass of the circular cylinder of water C_x of diameter equal to the depth of the body at cross-section S_x, while moving in the transverse z direction.

The transverse force per unit length is now given by,

$$L(x,t) = -\rho \left(\frac{\partial}{\partial t} + U_\infty \frac{\partial}{\partial x} \right) (v(x,t)A(x)), \tag{2}$$

whose integral along the length of the fish must balance the rate of change of lateral momentum. Similarly the yaw moment about the spanwise axis must balance the rate of change of angular momentum. This forms the basis for Lighthill's 'recoil correction' principle, which requires that for a prescribed backbone displacement, an imbalance in force and moment can be corrected with a recoil translation and rotation about the centre of mass. This is the basic concept used to analyse turning and is examined in more detail in Section 3.

An extension of 'elongated-body' theory for large amplitude motion assumes that the body is inextensible, with the independent Lagrangian variable s indicating position along the body from nose ($s = 0$) to tail ($s = 1$). The inextensibility condition is represented by,

$$\left(\frac{\partial h_x}{\partial s} \right)^2 + \left(\frac{\partial h_z}{\partial s} \right)^2 = 1. \tag{3}$$

Spatial integration of Eq. (3) gives $h_x(s, t)$. For small amplitude motion, x and s are interchangeable.

2.2. Numerical panel methods

Current computational capabilities now permit numerical solutions to fish swimming problems. These may then be compared with the analytical methods. Cheng et al. [7] considered an infinitesimally thin waving plate modelled as a network of rectangular vortex rings. The wake was modelled as a spanwise row of rings shed at each time step so as to satisfy the Kutta condition. Hill [9] extended Cheng's method for large amplitudes. (For details on vortex methods see [19].)