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Extended model for Richtmyer–Meshkov mix

Karnig O. Mikaelian *

Lawrence Livermore National Laboratory, Livermore, CA 94551, United States

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ABSTRACT

Mixing between two adjacent fluids has important consequences for inertial fusion capsule implosions and for supernova explosions. Consider an interface with small, random perturbations between two fluids of densities ρ_A and ρ_B. When a shock passes through the interface, those perturbations grow and the two fluids begin mixing in a process called Richtmyer–Meshkov (RM) mix, in analogy with Rayleigh–Taylor (RT) mix generated when the system undergoes a constant acceleration. Around the interface a time-dependent mixing width h evolves from the initial value h_0 and grows very large for strong shocks. If the interface sees a second shock, also called a reshock, the mixing is again intensified. In this paper we examine four RM experiments on shock-generated turbulent mix and find them to be in good agreement with our earlier simple model in which the growth rate $\dot{h}$ of the mixing layer following a shock or reshock is constant and given by $2\alpha A\Delta v$. Here A is the Atwood number $(\rho_B-\rho_A)/(\rho_B+\rho_A)$, Δv is the jump in velocity induced by the shock or reshock, and α is the constant measured in RT experiments: $\alpha^{bubble} \approx 0.05-0.07$, $\alpha^{spike} \approx (1.8-2.5)\alpha^{bubble}$ for $A \approx 0.7-1.0$. We then extend the model to $t > t*$ or, equivalently, $h > h*$ when the growth rate begins to decay and exhibit $h \sim t^{\theta}$ behavior. We ascribe this changeover to loss of memory of the *direction* of the shock or reshock, signaling the transition from highly directional to isotropic turbulence. In the simplest extension of the model, h^*/h_0 is independent of Δv and depends only on A: $h^*/h_0 \approx 2.5-3.5$ for $A \approx 0.7-1.0$.

1. Introduction

Hydrodynamic instabilities between two fluids and in particular Rayleigh–Taylor (RT) [1,2] and Richtmyer–Meshkov (RM) [3,4] instabilities have acquired new importance since the proposal [5] to use inertial confinement fusion to achieve thermonuclear burn [6]. In astrophysics they challenge our ability to explain certain phenomena such as supernova explosions [7]. These hydrodynamic instabilities cause the two fluids to interpenetrate and mix. Experimental, numerical and theoretical efforts continue to shed light on these complex, yet basic processes [8].

The original works [1–4] on RT and RM instabilities were naturally limited to the single-scale linear regime. For RT, perturbations of amplitude η and wavelength λ grow at the interface between two fluids of densities ρ_A and ρ_B under a constant acceleration $\vec{g}$ directed from A to B with $\rho_A < \rho_B$. For RM, perturbations grow if a shock passes from A to B or B to A. In the latter case the growth is preceded by a phase reversal. The linear regime for both instabilities is limited to $\eta \ll \lambda$.

As the amplitude grows it enters the nonlinear regime $\eta \geq \lambda$ and slows down but continues to grow. There is a vast and growing literature on nonlinear evolution that we forgo except to mention that due to the difficulty of nonlinear equations several models have been developed, of which we cite only Layzer's original work [9]. Its descendants are too numerous to report and not quite germane to the subject at hand, which is turbulent mix, a topic even more challenging: multi-wavelength initial perturbations, shocked or accelerated, evolving into turbulence. There are no expectations for an exact, first-principles description of turbulent mix anytime soon and therefore the development of models is even more requisite.

In Section 2 we review briefly the experiments on RT and RM mix, with emphasis on the latter. In Section 3 we present a quantitative comparison of our model with a recent RM experiment on reshocks where the interface is shocked twice. In Section 4 we extend our earlier model and apply it to the early and late-time evolution of both shocked and reshocked interfaces. Conclusions, future work, and suggestions for new experiments are presented in Section 5.

2. RT & RM experiments

We believe the first experiments on RT mix were those of Read [10], guided and supported by the numerical simulations of Youngs [11]. The mix is initiated by a multitude of wavelengths, some having amplitudes in the linear $\eta_i < \lambda_i$ and others in the nonlinear $\eta_i > \lambda_i$ regime, a combination that we call "random" for short. The resulting evolution, a very brief time after the start of

* Tel.: +1 925 422 5449.
E-mail address: mikaelian1@llnl.gov.

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the acceleration, was strikingly simple:

$$h = \alpha A g t^2, \quad (1)$$

where h is the mixing width, α a constant, and A is the Atwood number $(\rho_B - \rho_A)/(\rho_B + \rho_A)$. In our notation h stands for h^b or h^s and α for the corresponding α^b or α^s, with h^b (h^s) referring to the mixing-layer width on the bubble (spike) side, i.e., the penetration depth of the light (heavy) fluid into the other. The experiments [10] were driven by rockets and hence are often referred to as "rocket-rig" experiments. Initial conditions $h_0 \equiv h(t = 0)$ were not measured except to note that they were small and, as is clear from Eq. (1), they did not influence the growth of $h(t)$, a fact often referred to as "loss of memory of initial conditions". Only h^b was measured with $\alpha^b \approx 0.07$ constant for a large range of Atwood numbers.

Several subsequent experiments, of which we mention only a couple, confirm the above picture. "Water channel" [12] experiments reported $\alpha^b \approx \alpha^s \approx 0.07$. These were low-$A$ experiments and therefore it is expected that $h^b \approx h^s$. "LEM" (Linear Electric Motor) experiments [13] reported $\alpha^b \approx 0.05$ and $\alpha^s \approx \alpha^b[(1+A)/(1-A)]^{0.33} = \alpha^b[\rho_B/\rho_A]^{0.33}$ for $A \le 0.8$. We know of no experiment reporting any large effect of initial conditions on the RT mixing rate and in fact efforts to reduce mixing by reducing h_0 (smoother initial surfaces) have been largely unsuccessful, and the principle of "independence from initial conditions" appears well established. There are models predicting smaller α's for h_0 below a threshold [14], but apparently this threshold is difficult to achieve experimentally.

The role of initial conditions on the RT mixing is actually a longstanding debated issue, intimately related to the universality of the growth-rate parameter α. More recent experiments at a lower Atwood number, $A \approx 0.2$, find somewhat lower α's in the 0.03–0.04 range [15]. Varying the initial conditions, Olson and Jacobs report that "The measured α values do not show a dependence on the initial perturbation amplitude", though there is some small dependence on the spectral content of the initial spectrum [15]. Lower α values are preferred in numerical simulations which also find some difference between a narrow-band and a wide-band initial spectrum. Recent simulations and a review can be found in Ref. [16].

Wide-band initial spectra with multi-wavelength perturbations evolve more readily into turbulent mix, while the narrow-band spectra appear to require some transitional period to develop turbulence. This is perhaps natural because in the extreme narrow-band limit, which is a single wavelength, the nonlinear amplitude evolves as $\eta \sim t$ compared with turbulent mix $h \sim t^2$ as in Eq. (1).

Turning to RM, the first model proposed [17]

$$h = 2\alpha A \Delta v t, \quad (2)$$

thus maintaining the principle of "independence from initial conditions" and providing a complete prediction for the mixing width: α is the same constant as measured previously in RT experiments and Δv is the jump in the velocity of the interface induced by the shock. An initial-mix-width h_0 can appear as an additive constant in Eqs. (1) and (2) for consistency, but for now we take $h_0 \ll h$.

Clearly, the uncertainties and issues concerning α discussed in the above paragraphs carry over to this model: If α depends on the initial spectrum or amplitude it will be the same here also, and Eq. (2) may fail in the mode-coupling regime (16). These questions have been much less studied in the RM case. As in the RT case, many experiments and more simulations will be necessary to settle the issue of "dependence on initial conditions". We expect the narrow-band/wide-band dichotomy to persist. Indeed, the extreme-narrow-band (i.e., single-λ) nonlinear amplitude evolves as $\eta \sim \ln t$ compared with the turbulent $h \sim t$ expected from Eq. (2).

The purpose of this study is to compare Eq. (2) and its extension with four RM experiments. First came the experiments of Vetter and Sturtevant [18]. Next were the experiments of Erez et al. [19], followed by Leinov et al. [20]. These were all gas/gas experiments in shock tubes, in contrast to a gas/liquid experiment reported recently by Shi et al. [21]. In this section we discuss the experiments briefly. A quantitative comparison of Eq. (2) with the experiments of Leinov et al. will be given in Section 3 and, after its extension, in Section 4.

2.1. Vetter and Sturtevant [18]

These were experiments in a large horizontal shock tube using air and SF6 as the light and heavy gases, respectively, separated by a thin membrane which breaks up when the incident shock passes from air to SF6. In addition, they used a pair of thin-wire-grids and placed the membrane in 3 different locations: (i) Before, i.e. upstream side, (ii) Between, or (iii) After, i.e. downstream side of the grids. Once such a composite interface was shocked the membrane was shattered. The shock then reached the endwall of the tube, reflected back, and reshocked the air/SF6 interface.

Experiments were carried out with different shock strengths as measured by the Mach number M_s of the incoming shock. To keep the reshocked interface within their diagnostic window, Vetter and Sturtevant had to vary the length L of the test section containing SF6.

Only the total mixing width $h^t \equiv h^b + h^s$ was measured. To compare with Eq. (2), Vetter and Sturtevant assumed $\alpha^b = \alpha^s \approx 0.07$ (only the rocket-rig experiments [10] had been reported at that time with $\alpha^b \approx 0.07$), hence the coefficient $2(\alpha^b + \alpha^s) \approx 0.28$ in Eq. (2) of Ref. [18].

The growth rates after the first shock depended on which of the 3 configurations was used, and the interface evolved differently from one configuration to another. However, when the reshock struck these different structures, a "universal" growth rate was measured. These experiments indicated that the post-reshock growth rate was largely independent of pre-reshock conditions and was in good agreement with Eq. (2), particularly at higher Mach numbers [18].

2.2. Erez et al. [19]

These experiments were also conducted in a horizontal shock tube but with a smaller cross section. Air/SF6 was used here also separated by a thin or thick membrane. The results were consistent with those of Vetter and Sturtevant: The mixing width after the first shock depended on the type of membrane used, but the growth after reshock appeared to be linear in time, largely independent of which membrane was used, and independent of h_-, the value of the mixing width just before reshock. No comparison with Eq. (2) was made.

2.3. Leinov et al. [20]

These experiments were carried out in the same shock tube as Erez et al. with several modifications. Since evolution after first shock was known to depend on the membrane, they used only a thin membrane to separate air from SF6 and focused on the reshock. Three different methods were used to study the evolution of the total mixing width after reshock.

The first method was the same one used by Vetter and Sturtevant: Scan over shock Mach numbers M_s. As M_s increased, so did the growth rate $\dot{h}$ after reshock.

The second method was a simple yet important variant of the Vetter and Sturtevant experiments: Change the length of the test section but *keep M_s the same*. Vetter and Sturtevant had changed the length L of their test section and M_s simultaneously to accommodate their diagnostics; Leinov et al. kept M_s the same and changed only L to delay the arrival of the reshock at the