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Risk estimation of infectious diseases determines the effectiveness of the control strategy

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ABSTRACT

Usually, whether to take vaccination or not is a voluntary decision, which is determined by many factors, from societal factors (such as religious belief and human rights) to individual preferences (including psychology and altruism). Facing the outbreaks of infectious diseases, different people often have different estimations on the risk of infectious diseases. So, some persons are willing to vaccinate, but other persons are willing to take risks. In this paper, we establish two different risk assessment systems using the technique of dynamic programming, and then compare the effects of the two different systems on the prevention of diseases on complex networks. One is that the *perceived* probability of being infected for each individual is the same (uniform case). The other is that the *perceived* probability of being infected is positively correlated to individual degrees (preferential case). We show that these two risk assessment systems can yield completely different results, such as, the effectiveness of controlling diseases, the time evolution of the number of infections, and so on.

1. Introduction

Recently, the outbreaks of Severe Acute Respiratory Syndrome (SARS) [1,2], Avian influenza [3,4], and Swine influenza (H1N1) [5,6] have posed great threats to the human population. Modeling the spread of epidemics is an important topic in understanding the impact of diseases and designing effective control strategies, and has therefore become a task of utmost importance and attracted a revival of interest from the scientific community. Classical mathematical approaches make simplifying assumptions about the patterns of disease-causing interactions among hosts. In particular, homogeneous-mixing models assume that all hosts have identical rates of disease-causing contacts [7]. However, many infectious diseases are diffused from individual to individual following a heterogeneous contact pattern between them. So the transmission of diseases in the human population can be conveniently abstracted as diseases propagate on complex networks with different structures. Examples include the web of human sexual contacts [8], the distribution of avian influenza [4], and so on. Therefore, the dynamics of epidemics on complex networks [9–18] and a wide variety of immunization or vaccination strategies, including targeted immunization [19], acquaintance immunization [20], ring immunization [21], etc., are investigated under the framework of complex networks.

Though the above immunization strategies have proven to be efficient in controlling the diseases under certain conditions, an often neglected factor is that many vaccinations are voluntary rather than mandatory (for example influenza vaccination [22] and smallpox in some countries [23]). Under a voluntary vaccination mechanism, individuals typically aim at increasing their own interests, so they will balance the cost of vaccination against the risks of infection to decide whether to vaccinate or not in the presence of infection. However, the decision on vaccination is highly dependent on the individuals' *perceived* risk of the diseases, which is in turn determined by many factors, such as the prevalence of diseases, the transmission rate of diseases, the duration of diseases, and so on [22–29]. Thus, in this paper, the dynamics of infectious diseases on complex networks under the voluntary vaccination mechanism is investigated. Meanwhile, the effects of these factors on the *perceived* risk of diseases are established by a dynamic programming method in this paper.

Furthermore, to compare the effects of risk estimation systems on controlling the spread of epidemics on complex networks, we study two different cases: for the first case, we assume that the *perceived* risk of being infected for each susceptible individual on network is the same (uniform case), that is, each susceptible individual estimates the *perceived* risk of infection only depending on the prevalence of infection, not on its own degree/immediate

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neighbors at each time step. In the second case, we assume individuals are more rational, so the more links/neighbors they have, the higher the probability of being infected. As a result, the *perceived* risk of infection is not only proportional to the prevalence of the disease but also on the individual's degree/immediate neighbors (preferential case). Interestingly, even though there is only a small discrepancy between the two cases, completely different results are observed. For example, for the uniform case, the effect of voluntary vaccination on a scale-free network is worse than on random network. For the preferential case, however, the opposite occurs.

2. Model

In this paper we adopt the SIS epidemiological model to investigate the role of voluntary vaccination. In the SIS model, at each time step, each susceptible (S) node is infected with transmission rate β if it is connected to an infected (I) node. Meanwhile, the infected node recovers and returns to the susceptible state again with probability μ.

When facing an impending infectious disease, each susceptible individual has to decide whether to vaccinate or not by weighing the *perceived* risk of infection against the cost of vaccination. We assume that the *perceived* risk function for susceptible individuals comes from several aspects: the perceived prevalence of the disease estimated by individuals themselves, the transmission rate β, and the duration time of the disease $\tau = 1/\mu$. Furthermore, we assume that individuals are forward-looking, and they discount future wealth by a discount factor $\delta \in [0, 1)$. The discount factor represents how much weight an individual places on the future when deciding what action to take [30]. (The concept of such a discount is common in economic and accounting fields when computing the net present value of an asset. In general, the idea is that value at some distant time in the future has less utility than an equivalent value now, i.e., a present value u becomes $u\delta^t$ after t time periods.)

Suppose that each individual has the same initial wealth u, and if the individual is infected then his/her wealth is $u-c$, here $c > 0$ indicates the cost of infection. We assume that each individual i estimates the prevalence of disease in a uniform way

$$\theta_i = r_i I(t), \tag{1}$$

where r_i is uniform distribution in [0, 1], describing the imperfect information about the disease and the diversity of individuals, and $I(t)$ is the proportion/density of infection among the total population.

Denote f_r^U (f_r^P) and f_v^U (f_v^P) as the *perceived* risk function and cost of vaccination for the uniform case (preferential case), respectively. To reflect the optimal behavior of susceptible individuals, in this work we use the technique of dynamic programming [31] (see a brief introduction in the Appendix) to deduce the *perceived* risk functions f_v^U for the uniform case and f_v^P for the preferential case.

Let $V_i(S)$ and $V_i(I)$ be the value functions of individual i evaluated in the susceptible state and the infected state, respectively. So for the uniform case, $V_i(S)$ satisfies the following Bellman equation [30,32]

$$V_i(S) = \max\{u - f_v^U + \delta V_i(S), u + \delta[(1-\beta\theta_i)V_i(S) + \beta\theta_i V_i(I)]\}. \tag{2}$$

The first term and second term in the brackets of Eq. (2) are the individual i's benefit from vaccination and from non-vaccination, respectively. Moreover, $(1-\beta\theta_i)V_i(S)$ is the benefit of i escaping from infection though it takes risky behavior, and $\beta\theta_i V_i(I)$ is the benefit of being infected because of the risky behavior. The value function of i evaluated in infected state $V_i(I)$ is given as

$$V_i(I) = \sum_{t=0}^{\tau-1} \delta^t (u-c) + \delta^\tau V_i(S) = (u-c)\frac{1-\delta^\tau}{1-\delta} + \delta^\tau V_i(S). \tag{3}$$

To obtain the maximum value of $V_i(S)$, we first establish the following two equations from Eqs. (2) and (3)

$$\begin{cases} V_i(S) = u - f_v^U + \delta V_i(S), & \text{(a)} \\ V_i(S) = u + \delta\Big[(1-\beta\theta_i)V_i(S) \\ \quad +\beta\theta_i\left((u-c)\frac{1-\delta^\tau}{1-\delta} + \delta^\tau V_i(S)\right)\Big]. & \text{(b)} \end{cases} \tag{4}$$

Solving Eq. (4)(a) and (b), one has

$$\begin{cases} V_i(S) = \frac{u - f_v^U}{1-\delta}, & \text{(a)} \\ V_i(S) = \frac{u + \delta\beta\theta_i(1-\delta^\tau)(u-c)}{(1-\delta)(1-\delta(1-\beta\theta_i) - \beta\theta_i\delta^{\tau+1})}. & \text{(b)} \end{cases} \tag{5}$$

Without loss of generality, by setting $u = 0$ and taking the maximum value of $V_i(S)$ in Eq. (5)(a) and (b), we have

$$V_i(S) = \max\left\{\frac{-f_v^U}{1-\delta}, \frac{-\delta\beta\theta_i(1-\delta^\tau)c}{(1-\delta)(1-\delta(1-\beta\theta_i) - \beta\theta_i\delta^{\tau+1})}\right\}. \tag{6}$$

From Eq. (6), we know that if

$$\frac{-f_v^U}{1-\delta} > \frac{-\delta\beta\theta_i(1-\delta^\tau)c}{(1-\delta)(1-\delta(1-\beta\theta_i) - \beta\theta_i\delta^{\tau+1})}$$
$$\Rightarrow f_v^U < f_r^U \triangleq \frac{\delta\beta\theta_i(1-\delta^\tau)c}{1-\delta(1-\beta\theta_i) - \beta\theta_i\delta^{\tau+1}}, \tag{7}$$

then vaccination is the preferred choice; otherwise, if $f_v^U \geq f_r^U$, non-vaccination is a better choice.

Remark 1. To facilitate the analysis, we assume that the efficiency of vaccination is 100% during the period of the vaccine's validity.

For the preferential case, we assume that individuals are more rational than the uniform case, i.e., susceptible individuals know that the more neighbors they have the greater their probabilities of being infected. As a result, the degree of each susceptible individual is considered in the preferential case. In this case, individual i's value function $V_i(S)$ satisfies

$$V_i(S) = \max\{u - f_v^P + \delta V_i(S), u + \delta[(1-\beta k_i\theta_i)V_i(S) + \beta k_i\theta_i V_i(I)]\}, \tag{8}$$

here k_i is the degree of node i, other parameters are the same as the uniform case.

Similar to the uniform case, for the preferential case, if

$$f_v^P < f_r^P \triangleq \frac{\delta\beta\theta_i k_i(1-\delta^\tau)c}{1-\delta(1-\beta\theta_i k_i) - \beta\theta_i k_i\delta^{\tau+1}} \tag{9}$$

susceptible individuals are willing to be vaccinated; otherwise, $f_v^P \geq f_r^P$ susceptible individuals prefer to take risks.

3. Main results

In this section, we use two different risk functions—Eq. (7) for the uniform case and Eq. (9) for the preferential case—to study their different effects on the dynamics of epidemics. Our main results are based on the scale-free BA network proposed by Barabási and Albert (BA) in 1999 [33]. We also use the random network [34] here as a comparison, when the effects of structure on the dynamics of epidemic are considered. Both of them have the same size