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Discrete set of kink velocities in Josephson structures: The nonlocal double sine–Gordon model

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HIGHLIGHTS

- We study a nonlocal model which describes a Josephson layered structure.
- There exists a discrete set of velocities for non-radiating fluxon propagation.
- Numerical modeling shows that these velocities appear spontaneously in dynamics.
- An asymptotical formula for these velocities is conjectured.

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ABSTRACT

We study a model of Josephson layered structure which is characterized by two peculiarities: (i) superconducting layers are thin; (ii) the current–phase relation is non-sinusoidal and is described by two sine harmonics. The governing equation is a nonlocal generalization of double sine–Gordon (NDSG) equation. We argue that the dynamics of fluxons in the NDSG model is unusual. Specifically, we show that there exists a set of particular constant velocities (called "sliding" velocities) for non-radiating stationary fluxon propagation. In dynamics, the presence of this set results in quantization of fluxon velocities: in numerical experiments a traveling kink-like excitation radiates energy and slows down to one of these particular constant velocities, taking the shape of predicted 2π-kink. We conjecture that the set of these stationary velocities is infinite and present an asymptotic formula for them.

1. Introduction

Since mid-1960s the sine–Gordon equation

$$\sin\varphi + \omega_J^{-2}\varphi_{tt} = \lambda_J^2\varphi_{xx} \tag{1}$$

has been recognized as the basic model for description of a long contact between two superconductors (Josephson junction, JJ). Here $\varphi = \varphi(x, t)$ is the phase difference of the order parameters in the superconducting banks, ω_J is the Josephson plasma frequency, and λ_J is the Josephson length. The first term on the left-hand side of Eq. (1) comes from the formula for supercurrent across the JJ

$$J(\varphi) = J_c \sin\varphi, \tag{2}$$

where J_c is the critical Josephson current.

In mid-1970s it was recognized that in some situations the second derivative term on the right-hand side of Eq. (1) should be corrected. In particular, this should be done if the London penetration depth λ_L becomes comparable with Josephson length λ_J. Then Eq. (1) must be replaced by an integral equation of the type

$$\sin\varphi(x, t) + \omega_J^{-2}\varphi_{tt}(x, t) = \lambda_J^2 \frac{\partial}{\partial x}\int_{-\infty}^{\infty} dx' G(x, x')\varphi_{x'}(x', t). \tag{3}$$

In the literature Eq. (3) has been called *nonlocal sine–Gordon equation* [1]. Explicit form of the kernel $G(x, x')$ depends on physical and geometrical properties of JJ. A survey of recent results in this field (sometimes called *nonlocal Josephson electrodynamics*) including a list of kernels which have been used in the literature can be found in [1]. It has been found that dynamics of vortex in nonlocal Josephson electrodynamics has essential peculiarities. This issue has been discussed in many papers [2–6].

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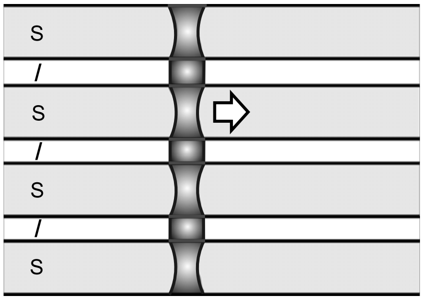

Fig. 1. Layered structure: S—superconducting layers, I—tunnel layers. A vortex formation moves from left to right.

Asides from isolated JJ, layered structures with Josephson interaction also have been discussed (see, for example, [7–16]). The principles of nonlocal Josephson electrodynamics can be applied also to such structures. In particular, in [17] it has been shown that structures consisting of alternating flat superconducting and tunnel layers (see Fig. 1) can be described by Eq. (3) with the Kac–Baker kernel $G(x,x') \sim e^{-\gamma|x-x'|}$,

$$\sin\varphi(x,t) + \omega_J^{-2}\varphi_{tt}(x,t) = \Lambda \frac{\partial}{\partial x}\int_{-\infty}^{\infty} dx' e^{-|x-x'|/\lambda_{\text{eff}}}\varphi_{x'}(x',t), \quad (4)$$

where

$$\Lambda = \frac{(\lambda_L + d)\lambda_J^2}{2\lambda_L\sqrt{(L+d)L}},$$

λ_L is the London penetration depth, $2d$ is the thickness of the each tunnel layer, $2L$ is the thickness of the each superconducting layer and

$$\lambda_{\text{eff}} \equiv \lambda_L\sqrt{\frac{L}{L+d}}.$$

In [17] Eq. (4) has been derived under the following assumptions: (i) superconducting layers are identical, (ii) tunnel layers are identical, (iii) vortex formation is symmetric, i.e. the phase difference and the magnetic field in all the JJs are identical, and (iv) the superconducting layers are thin when compared with the penetration depth of magnetic field into superconductors (see (A.2)). The model (4) possesses many fascinating features [2]. Among them are: existence of bound states of more than one flux quanta, quantization of velocities of these bound states and essential reducing of fluxon mobility.

Another motivation to modify Eq. (1) is caused by *non-sinusoidal character of current–phase relation*. Generally speaking, Eq. (2) represents the first term of the relation [18,19]

$$J(\varphi) = J_c \sin\varphi + J_2 \sin 2\varphi + J_3 \sin 3\varphi + \cdots. \quad (5)$$

In many situations $J_2 \gg J_k$, $k = 3, 4\ldots$. Therefore in many studies (see e.g. [20–23]) the current–phase relation has been assumed to be of the form

$$J(\varphi) = J_c \sin\varphi + J_2 \sin 2\varphi. \quad (6)$$

The sign of J_2 depends on a mechanism of suppression of superconducting state in electrodes. In SIS-type junctions $J_2 > 0$ due to suppression of superconductivity near the tunnel barrier by a supercurrent [24,25]. In SNINS and SFIFS junctions there exists another mechanism associated with the proximity effect [24–26], which may result in negative values of J_2. The equation for the phase difference φ in the case of current–phase relation (6) reads

$$\sin\varphi(x,t) + 2A\sin 2\varphi(x,t) + \omega_J^{-2}\varphi_{tt}(x,t) = \lambda_J^2\varphi_{xx}(x,t), \quad (7)$$

where $A \equiv J_2/2J_c$. This equation is *the double sine–Gordon equation* which has been widely discussed in both physical and mathematical literature.

In this paper we study the effect of non-sinusoidal current–phase relation on mobility of fluxons in Josephson structures with nonlocal electrodynamics. We present a model of Josephson structure consisting of alternating superconducting and tunnel layers. The structure is assumed to include infinitely many layers, in order to disregard boundary effects on the edges (see, e.g. [7–10]). It will be regarded that the current–phase relation is of the form (6). Also we assume that S-layers are thin and replace the second derivative term in Eq. (1) by a nonlocal term. Repeating the reasoning of [17] (the details of derivation can be found in Appendix A) and neglecting dissipation and bias terms we arrive at the equation

$$\sin\varphi(x,t) + 2A\sin 2\varphi + \omega_J^{-2}\varphi_{tt} = \Lambda\frac{\partial}{\partial x}\int_{-\infty}^{\infty} dx' e^{-|x-x'|/\lambda_{\text{eff}}}\varphi_{x'}(x',t). \quad (8)$$

The main output of our study can be formulated as follows. The properties of free propagation of Josephson vortices in the model (8) *essentially differ* from ones in (i) the traditional sine–Gordon model, (ii) the double sine–Gordon local model, Eq. (7), and (iii) the nonlocal sine–Gordon model, Eq. (4). In contrast to the models (i)–(iii), it has been found that there exist *a discrete set of constant velocities* (called in what follows "*sliding velocities*") of free stationary propagation of single fluxon without any radiation. The shapes of fluxons corresponding to different sliding velocities are *nearly the same*; the difference takes place in the asymptotics of the "tails" of these vortices. Moreover, the radiationless regime of propagation *is asymptotical* for some class of kink-shaped initial data.

The paper is organized as follows. In Section 2 we remind the concept of embedded soliton which is one of the basic concepts in the rest of the paper. In Section 3 we transform Eq. (8) into a dimensionless equation which depends on two external parameters, λ and A. We call it *nonlocal double sine–Gordon equation* and discuss some of its general features. In Section 4 we argue that fluxons in this model behave as embedded solitons and describe a phenomenon of *quantization* of velocities for stationary fluxon propagation. Specifically, we present the dependences of these *sliding* velocities on the parameter λ and give an asymptotical formula for them. In Section 5 we report on results of numerical simulation for fluxon evolution. We show that velocities of this set appear naturally in dynamics. Section 6 contains summary and discussion. Some physical details, including the derivation of the basic nonlocal equation (8), are postponed in Appendix A. Appendix B includes a statement from [27] which we use to describe asymptotical properties for the set of sliding velocities.

2. Sliding velocities and the concept of "embedded solitons"

The model (8) is dispersive and nonlinear. In general, in dispersive nonlinear systems *nonlinear modes* and *small-amplitude linear excitations* may coexist. Typically, there are some bounds for velocities of propagation for both linear and nonlinear excitations. If a velocity of nonlinear mode falls into the band of velocities of linear excitations, then the interaction with linear waves may cause emission of Cherenkov radiation, possible slowing down and stopping of the nonlinear wave. In context of Josephson electrodynamics, the phenomenon of Cherenkov radiation has been discussed, for instance, in [14,16,28–30] for a stacked JJs and in [31] for a nonlocal model of long JJ.

However, some isolated values of velocities for stationary propagation of nonlinear modes may exist, when no Cherenkov radiation occurs. These velocities are called *sliding* velocities. Discrete