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Aperiodic dynamics in a deterministic adaptive network model of attitude formation in social groups

Jonathan A. Ward [a,*], Peter Grindrod [b]

[a] School of Mathematics, University of Leeds, Leeds, LS2 9JT, UK

[b] Mathematical Institute, University of Oxford, Oxford, OX2 6GG, UK

HIGHLIGHTS

- A dynamical systems model of attitude formation in groups of interacting agents is proposed.
- Agents' states and the dynamic interaction network coevolve deterministically.
- Attitudinal dynamics are driven by an activator–inhibitor system.
- Linear stability analysis and numerical simulations are presented.
- The interplay between Turing instability and the evolving network gives rise to chaotic dynamics.

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ABSTRACT

Adaptive network models, in which node states and network topology coevolve, arise naturally in models of social dynamics that incorporate homophily and social influence. Homophily relates the similarity between pairs of nodes' states to their network coupling strength, whilst social influence causes coupled nodes' states to convergence. In this paper we propose a *deterministic* adaptive network model of attitude formation in social groups that includes these effects, and in which the attitudinal dynamics are represented by an activator–inhibitor process. We illustrate that consensus, corresponding to all nodes adopting the same attitudinal state and being fully connected, may destabilise via Turing instability, giving rise to aperiodic dynamics with sensitive dependence on initial conditions. These aperiodic dynamics correspond to the formation and dissolution of sub-groups that adopt contrasting attitudes. We discuss our findings in the context of cultural polarisation phenomena.

1. Introduction

In an adaptive network, the evolution of the states of nodes in the network coevolve with the network topology [1]. Adaptive network models have been proposed to describe a range of phenomena, including synchronisation [2–4], epidemics [5,6], cooperation [7–9] and opinion dynamics [10–16]. See Gross and Blasius [1] for a review. As with most complex systems, the usual paradigm is to describe the model dynamics via a stochastic process. In this paper we take an alternative approach, using an adaptive network described by a *deterministic* continuous-time dynamical system, which we use as a model for attitude formation in social groups. Earlier studies of deterministic adaptive network models have focused on the synchronisation of systems of coupled oscillators [17,18], although discrete-time models of opinion dynamics have also been proposed [19,20]. Part of our goal is to illustrate how techniques from nonlinear dynamics may be used to study adaptive networks in a way that compliments the typical statistical physics approach [21].

In this paper we consider a model of attitude formation in social groups, where the attitude of each node, or agent, in the system is described by a vector of states and the interaction patterns between agents are governed by an evolving weighted network. Our model incorporates two key behavioural mechanisms:

1. *Social influence.* This reflects the fact that people tend to modify their behaviour and attitudes in response to the opinions of others [22–26]. We model social influence via diffusion: agents adjust their state according to a weighted sum (dictated by the

* Corresponding author. Tel.: +44 1133435157.
E-mail address: j.a.ward@leeds.ac.uk (J.A. Ward).

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evolving network) of the differences between their state and the states of their neighbours.

2. *Homophily*. This relates the similarity of individuals' states to their frequency and strength of interaction [27]. Thus in our model, homophily drives the evolution of the weighted 'social' network.

A precise formulation of our model is given in Section 2. Social influence and homophily underpin models of social dynamics [21], which cover a wide range of sociological phenomena, including the diffusion of innovations [28–32], complex contagions [33–36], collective action [37–39], opinion dynamics [19,20,40,10,11,13,15,41,16], the emergence of social norms [42–44], group stability [45], social differentiation [46] and, of particular relevance here, cultural dissemination [47,12,48].

Combining the effects of social influence and homophily naturally gives rise to an adaptive network, since social influence causes the states of agents that are strongly connected to become more similar, while homophily strengthens connections between agents whose states are already similar.[1] It is surprising then that the feedback between homophily and social influence does not necessarily lead to consensus or 'monoculture' [47], where all nodes have identical states and are fully connected. Instead, cultural polarisation may occur: equilibria in which groups of nodes have identical states, but several different groups exist. Typically, cultural polarisation arises from the creation of 'structural holes' [51], which at its extreme leads to fragmentation of the network [15,16]. However, cultural polarisation is not necessarily stable when there is 'cultural drift', i.e. small, random perturbations or noise, which drives the system towards monoculture [52]. Since diversity and even polarisation of opinions are observed in society [47,48], there have been a number of attempts to develop models with polarised states that are stable in the presence cultural drift [12,48], but this is still an open problem [21].

In this paper, we investigate whether a general class of activator–inhibitor processes on an adaptive network can give rise to polarisation of attitudes. While the resulting dynamics illustrate that such systems are interesting in their own right, they are also perhaps a natural choice in the context of *sub-conscious* attitude formation. Neuropsychological evidence suggests that the *activation* of emotional responses and the *regulation* of inhibitions are associated with different parts of the brain [53]. This has led psychologists to develop theories in which various personality traits (such as extroversion, impulsivity, neuroticism and anxiety) form an independent set of dimensions along which different types of behaviour may be excited or regulated [54–56]. There is also substantial evidence that such automated and sub-conscious processes play an important role in evaluations and judgements [57]. Thus while it may be extremely difficult to perform empirical measurements on which models of sub-conscious attitude formation may be based, such processes almost certainly influence what we perceive to be conscious decision making.

One of the benefits of our dynamical systems formulation is that we are able to analyse the stability of the consensus equilibrium, and in Section 2 we show that Turing instability can arise. Furthermore, in Section 3 we illustrate, via numerical simulations, that the tension between Turing instability and the coevolution of the social network and attitudinal states gives rise to aperiodic dynamics that have a sensitive dependence on initial conditions. These dynamics correspond to the formation and dissolution of sub-groups that adopt distinct, non-equilibrium, attitudinal states. In Section 4 we discuss the transient patterns we observe in the context of cultural polarisation observed in other models.

[1] Note, however, that differentiating between the effects of homophily and social influence in observational settings may be very difficult [49,50].

2. A deterministic model of attitude formation

In this section we give a precise description of our adaptive network model of sub-conscious attitude formation in social groups. This model consists of two sets of coupled ordinary differential equations, one to describe the dynamics of agents' attitudinal states and the other to describe the evolution of the coupling strengths between nodes.

Consider a population of N identical individuals (nodes/agents/actors), each described by a set of M real attitude state variables that are continuous functions of time t. Let $\mathbf{x}_i(t) \in \mathbb{R}^M$ denote the ith individual's attitudinal state. In the absence of any influence or communication between agents we assume that each individual's state obeys an autonomous rate equation of the form

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i), \quad i = 1, \ldots, N, \tag{1}$$

where $\mathbf{f}$ is a given smooth field over $\mathbb{R}^M$, such that $\mathbf{f}(\mathbf{x}^*) = 0$ for some $\mathbf{x}^*$. Thus (1) has a uniform population equilibrium $\mathbf{x}_i = \mathbf{x}^*$, for $i = 1, \ldots, N$, which we shall assume is locally asymptotically stable. As discussed in the Introduction, we shall more specifically assume that (1) is drawn from a class of activator–inhibitor systems [58–60].

Now suppose that the individuals are connected up by a dynamically evolving weighted network. Let $A(t)$ be the $N \times N$ weighted adjacency matrix for this network at time t, with the ijth term, $A_{ij}(t)$, representing the instantaneous weight (frequency and/or tie strength) of the mutual influence between individual i and individual j at time t. Throughout we assert that $A(t)$ is symmetric, contains values bounded in $[0, 1]$ and has a zero diagonal (no self influence). We extend (1) and adopt a first order model for the coupled system:

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) + D \sum_{j=1}^{N} \left(\mathbf{x}_j - \mathbf{x}_i\right) A_{ij}(t), \quad i = 1, \ldots, N. \tag{2}$$

Here D is a real, diagonal and non-negative matrix containing the maximal transmission coefficients (diffusion rates) for the corresponding attitudinal variables between neighbours. Thus some of the attitude variables may be more easily or readily transmitted, and are therefore influenced to a greater extent by (while simultaneously being more influential to) those of neighbours. Note that $\mathbf{x}_i = \mathbf{x}^*$, for $i = 1, \ldots, N$, is also a uniform population equilibrium of the augmented system.

Let $\mathbf{X}(t)$ denote the $M \times N$ matrix with ith column given by $\mathbf{x}_i(t)$, and $\mathbf{F}(\mathbf{X})$ be the $M \times N$ matrix with ith column given by $\mathbf{f}(\mathbf{x}_i(t))$. Then (2) may be written as

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}) - D\mathbf{X}\varDelta. \tag{3}$$

Here $\varDelta(t)$ denotes the weighted graph Laplacian for $A(t)$, given by $\varDelta(t) = \mathrm{diag}(\mathbf{k}(t)) - A(t)$, where $\mathbf{k}(t) \in \mathbb{R}^N$ is a vector containing the degrees of the vertices ($k_i(t) = \sum_{j=1}^{N} A_{ij}(t)$). Eq. (3) has a rest point at $\mathbf{X} = \mathbf{X}^*$, where the ith column of $\mathbf{X}^*$ is given by $\mathbf{x}^*$ for all $i = 1, \ldots, N$.

To close the system, consider the evolution equation for the ijth edge, $A_{ij}(t)$, given by

$$\dot{A}_{ij} = \alpha A_{ij}(1 - A_{ij})(\varepsilon - \phi(|\mathbf{x}_i - \mathbf{x}_j|)). \tag{4}$$

Here $\alpha > 0$ is a rate parameter; $\varepsilon > 0$ is a homophily scale parameter; $|\cdot|$ is an appropriate norm or semi-norm; and $\phi : \mathbb{R} \to \mathbb{R}$ is a real function that incorporates homophily effects. We assume that $\phi(|\mathbf{x}_i - \mathbf{x}_j|) \geq 0$ and that ϕ is monotonically increasing with $\phi(0) = 0$. Note that the sign of the differences held in $\varepsilon - \phi(|\mathbf{x}_i - \mathbf{x}_j|)$ controls the growth or decay of the corresponding coupling strengths. The matrix $A(t)$ is symmetric, so in practice we only need to calculate the super-diagonal terms. The nonlinear "logistic growth"-like term implies that the weights remain in $[0, 1]$, while we refer to the term $\varepsilon - \phi(|\mathbf{x}_i - \mathbf{x}_j|)$ as the *switch* term.