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Frequency down-conversion using cascading arrays of coupled nonlinear oscillators

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ABSTRACT

A novel coupling scheme using $M \geq 2$ arrays of coupled nonlinear elements arranged in a specific configuration can produce multifrequency patterns or a frequency down-converting effect on an external (*input*) signal. In such a configuration, each array contains $N \geq 3$ nonlinear elements with similar dynamics and each element is coupled unidirectionally within the array. The subsequent arrays in the cascade are coupled in a similar fashion except that the coupling direction is arranged in the opposite direction with respect to that of the preceding array. Previous theoretical work and numerical results have already been reported in [P. Longhini, A. Palacios, V. In, J. Neff, A. Kho, A. Bulsara, Exploiting dynamical symmetry in coupled nonlinear elements for efficient frequency down-conversion, Phys. Rev. E 76 (2007) 026201]. This paper is focused on results of experiments implemented on two distinct systems: the first system is fabricated using discrete component circuits to approximate an overdamped bistable Duffing oscillator described by a quartic potential system, and the second system is built in a microcircuit, where the nonlinearity is described by a hyperbolic tangent function, with the option of applying an external signal to investigate resonant effects. In particular, the circuit implementations for each case use $M = 2$ arrays, but their voltage oscillations already demonstrate that the frequency relations between each of the successive arrays decrease by a rational factor, conforming to earlier theoretical and numerical results for the general case containing M arrays. This behavior is important for efficient frequency down-converting applications which are essential in many communication systems where heterodyning is typically used and it involves multi-step processes with complicated circuitry.

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1. Introduction

The process of generating new frequencies from an original oscillatory signal, either up-converting or down-converting the incoming signal, has been traditionally of interest in physics and engineering with applications that include: sensitive optical detection, music synthesis, acoustic and optical resonators, amplitude modulation, satellite communications, image extraction, and phase-noise measurements [1–7]. In recent works, we developed the theoretical foundations, and the first experimental demonstration, of an innovative technique for frequency up-conversion [8], while later, we extended the theory to include frequency down-conversion [9,10]. The underlying principle of this new technique is *symmetry* [11]. For instance, we have shown that symmetry-breaking Hopf bifurcations in a network of two arrays, each with N oscillators, possessing $\mathbf{Z}_N$-symmetry (cyclic group of permutations of N objects) can lead to one array oscillating in synchrony but at N times the frequency of the other array. We emphasize that the multifrequency effect is significantly different from that of subharmonic and ultraharmonic motion generated as is described by Hale and Gambill [12] and later by Tiwari and Subramanian [13]. In our case, the multifrequency behavior arises from the mutual interaction of the arrays via Hopf bifurcations. A distinctive feature of this approach is the model independent nature of its foundations, so it can be readily applicable to a wide range of dynamical classes and systems. Also important is that the approach can lead to various frequency up/down-conversion ratios in a single-step process using the dynamical behaviors as opposed to the multi-step process typical of heterodyning/superheterodyning methods which involve complicated circuitries and high precision, stable oscillators.

In this work, we show that a frequency down-conversion effect can be realized in physical systems comprising electronic circuits in full agreement with previous theoretical work [10]. The frequency down-conversion effect is achieved through a cascade of

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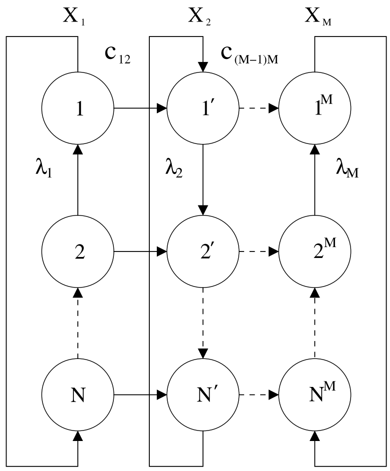

Fig. 1. Generalized network configuration of M arrays with N oscillators per array. Each array is coupled with a preferred direction but that direction alternates from one array to the next. The λ_is are coupling strengths within array i while C_{ij} is the strength of coupling from array i to array j.

Table 1
Down-conversion ratios between the frequencies of the X_1-array, ω_{X_1}, and X_2-array, ω_{X_2}, for a network of two coupled arrays interconnected as is shown in Fig. 1. k is a positive integer.

Number of cells	$\omega_{X_1}/\omega_{X_2}$			
3	2	5	$\cdots$	$3k-1$
5	4	9	$\cdots$	$5k-1$
7	6	13	$\cdots$	$7k-1$
9	8	17	$\cdots$	$9k-1$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
N	$N-1$	$2N-1$	$\cdots$	$Nk-1$

M arrays of oscillators, with N oscillators per array, as is illustrated in Fig. 1.

The efficiency in lowering the frequency from one array to the next can be used in many applications which require shifting the frequency down from high to low for ease of digitizing the signal using an available analog-to-digital converter suitable for the task, as many communication systems and others routinely do. In the absence of an external signal, and for the particular case of $M = 2$ symmetrical arrays with three elements $N = 3$ per array, the down-conversion ratio of the collective frequency of the first array can be $1/2$, $1/5$, or $1/11$. Table 1 shows a generalization of the down-conversion ratios for $M = 2$ arrays, including commensurate ratios, for various values of N. A more general result for M coupled arrays with N elements in each array obtained through various coupling topologies can be found in [10]. Therefore, the focus of this paper is on the experimental implementations to confirm the existence of those patterns for $M = 2$.

The experiments implemented here are based on two distinct systems. The first one consists of coupled overdamped Duffing oscillators, so the individual dynamics of each element in the array is governed by $\frac{dx}{dt} = ax - bx^3$, where $x(t)$ is the state variable at time t, and a and b are coefficients of the linear and nonlinear parts, respectively. This system is constructed using discrete component electronics capable of oscillating up to the kilohertz range. The second experiments consists of coupled bistable systems constructed from integrated circuits where the individual dynamics of each unit cell is described by $\tau\frac{dx}{dt} = -gx + I_s\tanh(c_s x)$, where τ is the time constant of the entire dynamics, g is a linear coefficient, and I_s and c_s are coefficients controlling the (*nonlinearity*) bistability of the dynamics. These two experimental systems are distinct in nature, which shows the generality and applicability of the frequency down-conversion method.

We have limited the experimental work in this paper to two cascading arrays in order to keep the complexity of the circuitry to a minimum while allowing us to demonstrate the phenomenology of frequency down-conversion. Analyses of the effects of noise on the signal and the system are important issues and we defer them to future work.

2. Experimental systems and results

A circuit version of Fig. 1, using a two-array cascade, was constructed using coupled overdamped Duffing oscillators. To simplify the use of the index notation for identifying the unit cells of each array, we will label the first array as the X-array and the second array as the Y-array from here on, and a single index i will suffice. Each array consists of subunits x_1, x_2, and x_3 for the X-array and y_1, y_2, and y_3 for the Y-array. The dynamics of the coupled arrays system are then described by

$$\begin{aligned}\dot{x}_i &= ax_i - bx_i^3 + \lambda_{xi}(x_{i+1})\\ \dot{y}_i &= ay_i - by_i^3 + \lambda_{yi}(y_{i+2}) + C_{xy}x_i,\end{aligned} \tag{1}$$

where $i = 1, 2, 3 \mod 3$, and x_i and y_i are the state variables of the ith subunit in the X-array and Y-array, respectively. λ_{xi} is the coupling coefficient of the ith subunit within the X-array, λ_{yi} is the coupling coefficient of the ith subunit within the Y-array, and C_{xy} is the cross-coupling from the X-array to the Y-array.

The circuit for each unit consists mainly of operational amplifiers, which act as summing–inverting amplifiers, and integrators, and produce the linear and nonlinear terms, which represent the dynamics of a subunit; see Fig. 2. Additional operational amplifiers are used for the unidirectional function of coupling between the oscillators within an array and the terms for the cross-coupling between the arrays, which connect the X-array to the Y-array as shown in the diagram of Fig. 1.

The complete diagram of the electronic network of the two arrays is given in Fig. 3. Careful attention is given to the selection of components in order to match the subunits in each array. Even after this is done, we still notice that they differ considerably. This is due to the availability of parts and to the difficulty in producing the nonlinear term, which in the electronic circuit is implemented as a piecewise linear function.

All values of the components used in the circuit are given in Table 2. They were selected through hardware simulations, via the SPICE program (an analog electronic circuit simulator), of a model of the circuit. Such hardware simulations are critical for minimizing the trial and error process of testing various resistor and capacitor values that set the coefficients a, b, λ_{xi}, λ_{yi}, and C_{xy} to the appropriate operational regime before fabricating the actual experimental system.

When the circuit is first powered up with a ± 5.0 V power supply, λ_{yi} is set to a value slightly below the critical value, λ_c, for a sustained oscillation. This critical coupling value was determined in [14] to be $\lambda_c = \frac{a}{2}$ when the arrays are decoupled from each other. From the circuit diagram, the linear coefficient a is determined to be $\frac{R_1}{R_3}$ which sets $\lambda_c = \frac{1}{12}$. Setting the coupling value is done by choosing the particular value in the feedback resistors, R_{cy}, where $\lambda_{cy} = \frac{R_5}{R_{cy}}$. As a result, all the unit cells in the array remain in stationary states. The X-array is set, with $\lambda_{xi} = \frac{R_5}{R_{cx}}$, to slightly greater than the critical value to initiate the oscillation in an out-of-phase pattern. It may take several trials to get to the correct pattern because the behavior is dependent on the initial condition of the coupled systems. In one instance a different pattern may show up where the oscillations of the unit cells are