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Reduced Navier–Stokes equations near a flow boundary

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Abstract

We derive a hierarchy of PDEs for the leading-order evolution of wall-based quantities, such as the skin-friction and the wall-pressure gradient, in two-dimensional fluid flows. The resulting *Reduced Navier–Stokes* (*RNS*) equations are defined on the boundary of the flow, and hence have reduced spatial dimensionality compared to the Navier–Stokes equations. This spatial reduction speeds up numerical computations and makes the equations attractive candidates for flow-control design. We prove that members of the RNS hierarchy are well-posed if appended with boundary-conditions obtained from wall-based sensors. We also derive the lowest-order RNS equations for three-dimensional flows. For several benchmark problems, our numerical simulations show close finite-time agreement between the solutions of RNS and those of the full Navier–Stokes equations.
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1. Introduction

1.1. Background and motivation

The approximation of Navier–Stokes flows near a no-slip boundary was apparently first discussed in detail by Perry and Chong [11], who developed a procedure for finding the Taylor coefficients of a velocity field expanded at a boundary point. By this procedure, one can construct velocity models that are polynomials in terms of the distance from the point of expansion. The models are dynamically consistent up to any desired order, but depend on properties imposed a priori on the velocity derivatives at the wall.

Danielson and Ottino [3] used the above procedure to construct a system of ODEs for the Taylor coefficients of a velocity field at a no-slip boundary point. The ODE system becomes finite-dimensional upon truncation of the Taylor expansion; Danielson and Ottino showed that even low-order truncations may lead to ODEs with a strange attractor, a hallmark of Eulerian turbulence.

Recently, Bewley and Protas [12] proposed a less restrictive Taylor-expansion of the velocity in terms of the normal distance from the boundary. For two-dimensional flows, this procedure yields a single-variable Taylor-expansion with coefficients depending on the location along the boundary. Bewley and Protas showed that under appropriate conditions, the expansion converges in the vicinity of the wall. In addition, for incompressible flows, all Taylor coefficients can be expressed in terms of time- and wall-tangential derivatives of the wall shear (skin friction) and the wall pressure.

With the availability of accurate skin-friction and pressure sensor-arrays, the results in [12] enable local velocity reconstruction from wall-based measurements. This offers a promising tool for practical flow control, where the impact of the controller must be evaluated from wall sensors. Feedback control, however, requires more than just an observation of the output: a model for the

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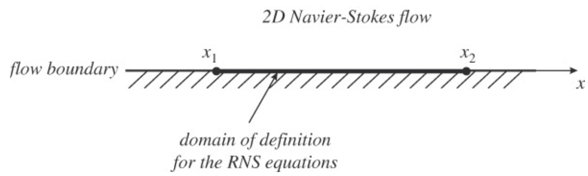

Fig. 1. Domain of definition for the RNS equations.

evolution of the flow is also crucial. The Bewley–Protas results offer hope that, at least near the wall, such models are reducible to depend only on the skin friction and wall pressure.

Further underlying the need for such reduced flow models, typical performance objectives in flow-control are often phrased directly in terms of skin friction and wall pressure, not velocity. Examples include pressure-recovery enhancement in diffusers and surface-drag reduction on submarines. The former aims to maximize the integral of the wall-pressure gradient; the latter to minimize the integral of the skin friction. In both cases, a qualitative prediction for the evolution of the underlying quantity is more beneficial than a highly accurate but complex numerical model.

1.2. Main results

Motivated by the above, here we study how the dynamics of wall-based quantities, such as the wall-shear τ and the wall-pressure gradient γ, can be modelled and predicted in two-dimensional Navier–Stokes flows. Our main result is a hierarchy of models, the *Reduced Navier–Stokes* (*RNS*) *equations*, that describe the evolution of the above quantities at different levels of accuracy. Since the RNS equations are defined on the flow boundary, they only have one spatial dimension, the wall coordinate x. This dimensional reduction results in computation times that are significantly shorter than those of direct Navier–Stokes simulations.

Solving the RNS equations requires updated boundary conditions for τ and γ at two x-locations, x_1 and x_2 (see Fig. 1). Thus, x_1 and x_2 must either be points with a priori known velocity derivatives (e.g., corner points), or must lie within distributed skin-friction and wall-pressure sensor arrays. In either case, the RNS equations provide qualitative prediction for the time-evolution of $\tau(x, t)$ and $\gamma(x, t)$ over the spatial interval $[x_1, x_2]$. The prediction necessarily deteriorates over time; solving the RNS equations over longer times therefore requires periodic re-initialization by sampling $\tau(x, t)$ and $\gamma(x, t)$ from sensors distributed over $[x_1, x_2]$.

We derive three members of the RNS hierarchy explicitly; these evolution equations are obtained from cubic, quartic, and quintic truncations of the Taylor expansion of the wall-tangential velocity component. We prove that these three RNS equations and all higher-order RNS systems are well-posed, i.e., admit unique solutions that depend continuously on the initial data. We also derive the lowest-order RNS equation for three-dimensional flows, and discuss the relevance of the two-dimensional RNS equations in select flow-control problems.

We present evidence for the accuracy of the RNS equations by comparing their numerical solution to classic solutions of the Navier–Stokes equations. These classic solutions include a viscous channel flow, the Blasius boundary layer solution, viscous flow near a stagnation point, and an oscillating flow over an infinite plate. We finally compare the direct numerical simulation of a lid-driven cavity flow to that of the RNS equations. In all cases, we observe close quantitative agreement on short to intermediate timescales, and qualitative accuracy over longer timescales.

2. RNS equations for two-dimensional flows

Consider the two-dimensional Navier–Stokes equations

$$\partial_t u + u_x u + u_y v = -\frac{1}{\rho} p_x + \nu \left(u_{xx} + u_{yy}\right),$$

$$\partial_t v + v_x u + v_y v = -\frac{1}{\rho} p_y + \nu \left(v_{xx} + v_{yy}\right), \qquad (1)$$

where $(u(x, y, t), v(x, y, t))$ is a velocity field satisfying the incompressibility condition

$$u_x + v_y = 0, \qquad (2)$$

and the no-slip boundary condition

$$u(x, 0, t) = v(x, 0, t) = 0 \qquad (3)$$

at the $y = 0$ boundary. In (1), $p(x, y, t)$ denotes the pressure, and ν and ρ are the kinematic viscosity and the density of the fluid.

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