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Interactions between two-dimensional solitons in the diffractive–diffusive Ginzburg–Landau equation with the cubic–quintic nonlinearity

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ABSTRACT

We report the results of systematic numerical analysis of collisions between two and three stable dissipative solitons in the two-dimensional (2D) complex Ginzburg–Landau equation (CGLE) with the cubic–quintic (CQ) combination of gain and loss terms. The equation may be realized as a model of a laser cavity which includes the spatial diffraction, together with the anomalous group-velocity dispersion (GVD) and spectral filtering acting in the temporal direction. Collisions between solitons are possible due to the Galilean invariance along the spatial axis. Outcomes of the collisions are identified by varying the GVD coefficient, β, and the collision "velocity" (actually, it is the spatial slope of the soliton's trajectory). At small velocities, two or three in-phase solitons merge into a single standing one. At larger velocities, both in-phase soliton pairs and pairs of solitons with opposite signs suffer a transition into a delocalized chaotic state. At still larger velocities, all collisions become quasi-elastic. A new outcome is revealed by collisions between slow solitons with opposite signs: they self-trap into persistent *wobbling dipoles*, which are found in two modifications — horizontal at smaller β, and vertical if β is larger (the horizontal ones resemble "zigzag" bound states of two solitons known in the 1D CGL equation of the CQ type). Collisions between solitons with a finite mismatch between their trajectories are studied too.

1. Introduction

Complex Ginzburg–Landau equations (CGLEs) constitute a vast class of models for the pattern-formation dynamics and spatiotemporal chaos in one- and multi-dimensional nonlinear media combining dissipative and dispersive/diffractive properties [1]. In particular, stable localized pulses ("dissipative solitons" [2]) can be supported by CGLEs that meet the obvious necessary condition of the stability of the zero background. This condition rules out the simplest cubic CGLE, whose one-dimensional (1D) variant admits well-known exact analytical solutions for solitary pulses [3]. The stability can be achieved in systems of linearly coupled equations, with one featuring linear gain and the other — linear loss [4]. In such a model, exact stable solutions for 1D solitons are available [5]. Another possibility is to use the CGLE with the cubic–quintic (CQ) combination of nonlinear terms. For the first time, the CGLE of the CQ type was introduced by Petviashvili and Sergeev [6] in the 2D form, with the intention to construct stable fully localized 2D states. In 1D, stable dissipative solitons of the CQ CGLE had been later studied in detail [7], including the analysis of two-soliton bound states [8,9]. Then, stable fundamental solitons [10–12] and localized vortices (alias spiral solitons) [12,13] have been found in 2D and 3D [14] models of the CQ-CGLE type, as well as in the complex Swift–Hohenberg equation with the CQ nonlinearity [15]. Such equations find their most significant physical realization as models of large-area laser cavities, where the CQ combination of the loss and gain is provided by the integration of linear amplifiers and saturable absorbers [16].

In most of the above-mentioned works [6,8,9,12–15,17], localized pulses and vortices were obtained as solutions to isotropic 2D equations. On the other hand, the CGLE which governs the spatiotemporal evolution of light in the large-area laser cavity is anisotropic, as it includes "diffusion" (the spectral filtering) acting only along the temporal variable. The existence of stable fully localized pulse solutions in the latter case suggests a possibility of the experimental creation of "light bullets", i.e., spatiotemporal optical solitons, in the cavities. In other physical contexts (unrelated to optics), anisotropy of the 2D CGLE was introduced in a different form, through unequal diffusion coefficients in the two perpendicular directions [18].

In Refs. [11], stable spatiotemporal dissipative solitons were found in the model of the laser-cavity type, based on the following normalized CGLE with the CQ nonlinearity:

$$iU_Z + \frac{1}{2}U_{XX} + \frac{1}{2}(\beta - i)\,U_{TT} = -\left[iU + (1 - i\gamma_1)\,|U|^2U + i\gamma_2|U|^4U\right]. \quad (1)$$

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Here, Z and X are the propagation and transverse coordinates in the cavity, and $T \equiv t - Z/V_0$ is, as usual, the reduced time, with t the physical time and V_0 the group velocity of the carrier wave. Term U_{XX} in Eq. (1) represents the transverse diffraction in the paraxial approximation, the coefficients accounting for the above-mentioned spectral filtering, Kerr nonlinearity, and background linear loss are all scaled to be 1, while β corresponds to the group-velocity dispersion (GVD). Usually, a necessary condition for the existence of temporal solitons is $\beta > 0$ [19], which implies the anomalous type of the GVD (in the present model, spatiotemporal solitons also tend to be more stable at $\beta > 0$ [11]). Further, positive coefficients γ_1 and γ_2 in Eq. (1) account for the cubic gain and quintic loss, respectively, which are characteristic features of CQ models. The third-order GVD was also taken into regard in Refs. [11], but this term is not considered here, as it does not essentially affect the results reported below. Because it combines the diffraction along X and effective diffusion along T, Eq. (1) is called the diffractive–diffusive CGLE [11].

Once 2D solitons are available, an issue of obvious interest is to explore collisions between them, provided that they are mobile, i.e., the equation is Galilean invariant. The 2D CGLE with no diffusion obviously satisfies this condition, allowing free motion of solitons or localized vortices in any direction. This property was used in Ref. [12] to study collisions between solitons in the isotropic CQ CGLE, as well as their motion in external potentials. It was concluded that collisions between fundamental solitons result in their quasi-elastic passage through each other (with a resultant *increase* of the relative velocity), or mutual destruction of the solitons, or their merger into a single 2D pulse. In the same model, collisions between vortices demonstrated a quasi-elastic rebound.

The laser-cavity model based on Eq. (1) features the Galilean invariance along the X-direction, which means that a moving solution can be generated from a quiescent one by the application of the Galilean boost corresponding to arbitrary "velocity" P (in fact, P is the tilt in the (X, Z) plane):

$$U(X, T, Z) \to \exp\left[\mathrm{i}\left(PX - P^2 Z/2\right)\right] U(X - PZ, T, Z). \tag{2}$$

This possibility suggests to consider collisions of 2D solitons in this model too. In this work, we report results obtained by means of systematic simulations of collisions between two and three solitons in the framework of Eq. (1). In the former case, both head-on collisions and those with a finite offset (*aiming distance*) between trajectories of the two solitons will be studied. In either case, we consider collisions between in-phase and out-of-phase 2D solitons (the latter means that they have opposite signs).

In Section 2, we report the results for two-soliton collisions, and in Section 3 — for interactions between three solitons. Outcomes of the collisions between two in-phase solitons include the quasi-elastic passage at large velocities, delocalization in the X-direction (merger into an expanding quasi-turbulent state) at intermediate velocities, and merger of slowly moving solitons into a single stable pulse. A major difference for collisions between out-of-phase solitons is that, at small velocities, they do not merge into a single pulse; instead, they may form a new localized object — a *wobbling dipole*, i.e., a robust bound state of two solitons with opposite signs, which feature persistent oscillations relative to each other in the spatial direction. Moreover, two different species of the wobbling dipoles are reported below, *horizontal* and *vertical* ones. In the latter case, the out-of-phase solitons, although they collide head-on, shift in opposite perpendicular directions (along the T-axis), and eventually form a dipole with a fixed vertical separation between them. Unlike the results for collisions between dissipative solitons in the 2D isotropic CGLE [12], in the present model we have never observed complete destruction (decay) of colliding solitons. For collisions with a finite aiming distance ΔT, we identify a critical value of ΔT which separates interactions and the straightforward passage. Three-soliton configurations feature either merger into a single pulse, or the transition into a delocalized chaotic state. In terms of the optical cavities, the various outcomes of the collisions offer possibilities for the use in all-optical data-processing schemes.

2. Two-soliton collisions

2.1. The numerical procedure

Eq. (1) was solved by means of the 2D split-step Fourier method with 256×256 modes and periodic boundary conditions in X and T, for the fixed size of the integration domain in both directions, $|X, T - 10| \leq 10$. The stepsize for the advancement in Z was 0.01. To generate the first stable 2D pulse boosted to velocity (tilt) P, cf. Eq. (2), an initial configuration was taken as

$$U_0(X, T) = \exp\left[-\left(X^2 + T^2\right)/4 + \mathrm{i}PX\right], \tag{3}$$

see the first panel in Fig. 2 below. The numerical integration of Eq. (1) led to quick self-trapping of the input pulse into a moving (tilted) dissipative soliton, which is an attractor of the model. The profile of the established soliton can be seen in the first panels of Figs. 3, 5 and 7.

Generic results for collisions between the solitons with velocities $\pm P$ can be adequately represented by fixing the cubic gain and quintic loss coefficients to be $\gamma_1 = 2.5$, $\gamma_2 = 0.5$, while varying P and GVD coefficient β. To generate diagrams presented below in Figs. 1 and 4, which display outcomes of the collisions, we changed β and P by small steps, the initial configuration for each simulation being a stable pulse produced by the simulations at the previous step.

2.2. Head-on collisions between in-phase solitons

Outcomes of collisions between two identical stable solitons, set by kicks $\pm P$ on the head-on collision course, are summarized in Fig. 1. Stable solitons exist only for $\beta \geq \beta_{\min} \approx -0.5$, which determines the left-hand edge of the diagram.

The simplest outcome of the collision is the straightforward quasi-elastic passage of the solitons through each other. We do not illustrate it by a separate picture, as it seems quite obvious; as well as in Refs. [11], the solitons keep the mutual symmetry after the quasi-elastic collision, and demonstrate some increase of "velocity" P (recall it is actually defined as the tilt of the soliton's trajectory in the (X, Z) plane). Moreover, running the simulations in the domain with periodic boundary conditions, we observed multiple quasi-elastic collisions between solitons. The solitons which emerge unscathed from the first collision survive indefinitely many repeated collisions as well.

With the decrease of the collision velocity, the quasi-elastic passage is changed by the delocalization. This means that two in-phase solitons, interacting attractively, merge into a single pulse, which, however, fails to self-trap into a standing soliton. Instead, it gives rise to a quasi-chaotic ("turbulent") state, that remains localized in the temporal direction (T), but features indefinite expansion along X, see a typical example in Fig. 2. This outcome may be explained by the fact the fused state has too much "intrinsic inertia", imparted by original velocities $\pm P$, which pushes the pulse to expand.

At still smaller values of P, the collision also gives rise to merger of the two solitons into a single pulse. However, in that case, the decrease of the above-mentioned "intrinsic inertia" allows the fused pulse to form a stable soliton, see a typical example in Fig. 3.