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Continuation of periodic solutions in the waveguide array mode-locked laser

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ABSTRACT

We apply the adjoint continuation method to construct highly accurate, periodic solutions that are observed to play a critical role in the multi-pulsing transition of mode-locked laser cavities. The method allows for the construction of solution branches and the identification of their bifurcation structure. Supplementing the adjoint continuation method with a computation of the Floquet multipliers allows for explicit determination of the stability of each branch. This method reveals that, when gain is increased, the multi-pulsing transition starts with a Hopf bifurcation, followed by a period-doubling bifurcation, and a saddle–node bifurcation for limit cycles. Finally, the system exhibits chaotic dynamics and transitions to the double-pulse solutions. Although this method is applied specifically to the waveguide array mode-locking model, the multi-pulsing transition is conjectured to be ubiquitous and these results agree with experimental and computational results from other models.

1. Introduction

High-power pulsed lasers are an increasingly important technological innovation. Their conjectured and envisioned applications, ranging from military devices and precision medical surgery to optical interconnection networks [1], have grown significantly over the past two decades. Such lasers are one of the few examples of a commercially viable photonics technology that are based fundamentally on nonlinear processes. As a result, mode-locking technologies have placed a premium on the engineering and optimization of laser cavities that are aimed at producing output pulses of tens to hundreds of femtoseconds with maximal peak powers in the kilowatt range and energies exceeding 10 nJ. Such technological demand has pushed mode-locked lasers to the forefront of commercially viable, nonlinear photonic devices. One of the most recently envisioned methods for generating stable mode-locking incorporates the intensity discrimination induced by the nonlinear mode-coupling properties in a waveguide array [2–6]. The waveguide array mode-locking produces robust mode-locking and displays the ubiquitous multi-pulsing transition instability [7,8] whereby an increase in the laser cavity energy above a given threshold causes a single-pulse per round trip to bifurcate to two pulses per round trip. This multi-pulsing transition dynamics is the primary focus of this manuscript.

Fig. 1 illustrates two possible mode-locking configurations in which the waveguide array provides the critical effect of intensity discrimination (saturable absorption) [1,9]. In Fig. 1(a), a linear cavity configuration is considered whereas in Fig. 1(b), a ring cavity geometry is considered. In either case, the waveguide array provides an intensity dependent pulse shaping by coupling out low intensity wings to the neighboring waveguides through a process called nonlinear mode-coupling.

Optical nonlinear mode-coupling (NLMC) is a well-established phenomenon that has been both experimentally verified [10–14] and theoretically characterized [15–17]. NLMC has been an area of active research in all-optical switching and signal processing applications using waveguide arrays [11–14], dual-core fibers [10,15,16], and fiber arrays [18,19]. It is only recently that the temporal pulse shaping associated with NLMC has been theoretically proposed for the passive intensity-discrimination element in a mode-locked fiber laser [2,3]. The models derived to characterize the mode-locking consist of two governing equations: one for the fiber cavity and a second for the NLMC element [2,3] (see Fig. 1). Although the two discrete components provide accurate physical models for the laser cavity, characterizing the underlying laser stability and dynamics is often analytically intractable. Thus, it is helpful to construct an averaged approximation to the discrete components model in order to approximate and better understand the mode-locking behavior. Indeed, the essence of Haus' master mode-locking theory [1] is approximating discrete elements with a continuous model. The same approach is used here to generate a continuous system of governing equations from a system that would, due to the inclusion of the waveguide array and Erbium fiber, include discrete effects [4,5].

Even with these continuous models, such as the waveguide array mode-locking model (WGAML) [2–5] used in this manuscript,

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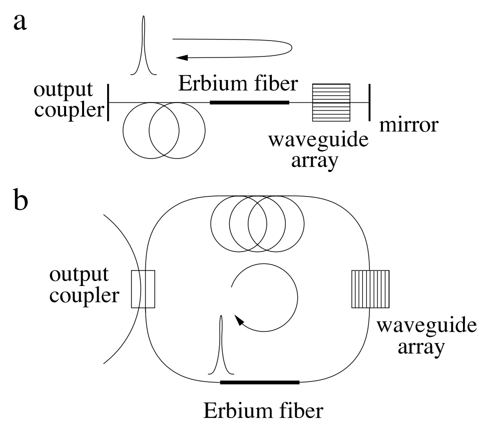

Fig. 1. Two possible laser cavity configurations that include nonlinear mode-coupling from the waveguide array as the mode-locking element. The fiber coupling in and out of the waveguide array occurs at the central waveguide as illustrated. Any electromagnetic field that is propagated into the neighboring waveguides is ejected (attenuated) from the laser cavity. In addition to the basic setup, polarization controllers, isolators, and other stabilization mechanisms may be useful or required for successful operation.

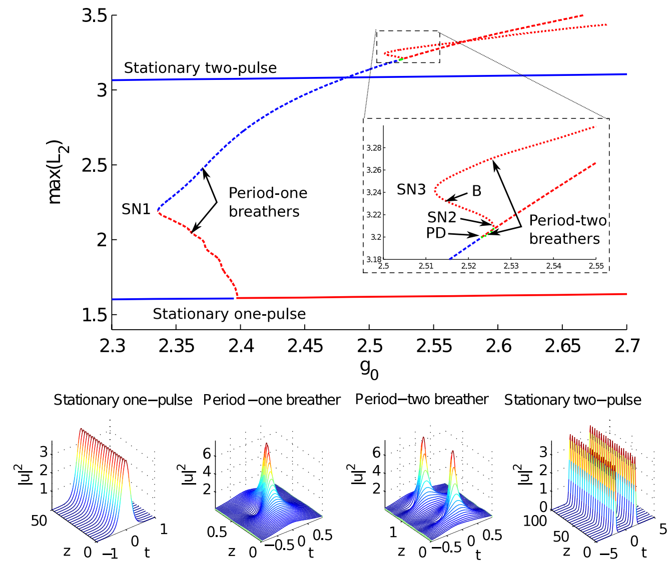

Fig. 2. (Color online) (Top) Bifurcation diagram including the branches of stationary one-pulse, stationary two-pulse, period-one breather, and period-two breather solutions. Branches in blue or green are linearly stable while branches in red are linearly unstable. Branches in solid lines are from stationary (constant amplitude) solutions while branches in dashed lines are z-periodic solutions. The green dashed lines represent period-two breathers and the blue lines period-one. Hopf, saddle–node, and period-doubling bifurcations are denoted by H, SN, and PD respectively. A fourth unknown bifurcation is indicated by B. (Bottom) Examples of the four qualitatively different solution behaviors – stationary one-pulse (single-pulse) solutions, period-one breathers, period-two breathers, and stationary two-pulse (double-pulse) solutions – observed during the multi-pulsing transition. The stationary two-pulse solutions can be treated as two non-interacting stationary one-pulse solutions.

an accurate characterization of the bifurcation structure had not been performed. Specifically, little was known about the branch of observed z-periodic breather solutions. A recent study towards qualitative understanding of the bifurcation process involves approximating the bifurcation sequence qualitatively using principal components in a low-dimensional reduction [6].

In this work, we use a hybrid numerical method, called the adjoint continuation method (ACM), that is able to calculate arbitrarily accurate solutions and perform a PDE bifurcation study. In particular, the method reveals the key aspects about and the complexity of the bifurcation structure of the multi-pulsing instability, an overview of which is shown in Fig. 2. The branches of solutions believed to be involved in the multi-pulsing transition can be separated into four qualitatively different types: stationary one-pulse (single-pulse) solutions, period-one breather solutions, period-two breather solutions, and stationary two-pulse (double-pulse) solutions with an example of each shown in the bottom of Fig. 2. The stationary one- and two-pulse solutions possess a constant amplitude with a linearly increasing phase. The period-one breather solutions have a z-periodic amplitude and are even functions over the entire period up to a translation in t. The period-two breather solutions are also z-periodic in amplitude, but they are neither even nor odd functions for the entire period.

By studying the stability of solution branches, we find that a subcritical-Hopf bifurcation occurs on the one-pulse solution branch, at the point labeled H in Fig. 2. This generates a branch of period-one breather solutions. This period-one (breather) branch first undergoes a saddle–node bifurcation (SN1) which is followed by a period-doubling bifurcation (PD). This period-doubling bifurcation creates the branch of period-two breather solutions. The period-two solution branch undergoes three bifurcations, labeled SN2, B, and SN3, and eventually leads to complex spatio-temporal (chaotic-like) behavior. Coexisting with these solution branches is a stationary two-pulse solution branch. This bifurcation diagram represents the underlying nonlinear phenomenon of the multi-pulsing transition dynamics. A detailed account of each solution branch and its complex transitions is developed herein. These results extend and justify the results of previous qualitative efforts in [6] and also have revealed new information about the source of symmetry breaking. Further our results hint at the mechanism for the onset of spatial–temporal disorder in the WGAML, and they are also consistent with recent experimental observations of the transition dynamics in laser cavities [20,21].

The paper is arranged as follows: Section 2 gives a brief overview of the governing averaged equations in the laser cavity. Section 3 develops the algorithm necessary for computing solution branches and following bifurcations to new paths of solutions. The bifurcation structure of the waveguide array mode-locked laser is given in Section 4. A brief summary and outlook for the method and the laser system is given in Section 5.

2. Governing equations

When placed within an optical fiber cavity, the pulse shaping mechanism of the waveguide array leads to stable and robust mode-locking [2,3]. In its simplest form, the nonlinear mode-coupling is averaged into the laser cavity dynamics [5]. Numerical simulations have shown that the fundamental behavior in the laser cavity does not change when considering more than five waveguides [5]. Further simplifications to the five waveguide model can be achieved by making use of the symmetric nature of the coupling and lower intensities in the neighboring waveguides [4]. The resulting approximate evolution dynamics describing the waveguide array mode-locking model (WGAML) is given by

$$\mathrm{i}\frac{\partial u}{\partial z} + \frac{D}{2}\frac{\partial^2 u}{\partial t^2} + \beta |u|^2 u + Cv + \mathrm{i}\gamma_0 u - \mathrm{i}g(z)\left(1 + \tau\frac{\partial^2}{\partial t^2}\right)u = 0 \tag{1a}$$

$$\mathrm{i}\frac{\partial v}{\partial z} + C(w + u) + \mathrm{i}\gamma_1 v = 0, \tag{1b}$$

$$\mathrm{i}\frac{\partial w}{\partial z} + Cv + \mathrm{i}\gamma_2 w = 0, \tag{1c}$$