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Controlling a complex dynamical network to attain an inhomogeneous equilibrium

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ABSTRACT

Many networks are made up of a few groups, with nodes in the same group having the same kind of function. In this work, the problem of controlling a complex dynamical network to attain an inhomogeneous equilibrium point is investigated, which means that nodes in the same group achieve the same equilibrium point as an isolated node, while different groups correspond to different equilibrium points. An open-loop constant control approach is first proposed to obtain the inhomogeneous equilibrium point of the network. Then, the feedback pinning control approach is applied to make the inhomogeneous equilibrium point asymptotically stable.

1. Introduction

Recently, there has been increasing research interest in networks with complex topologies and nonlinear node dynamics [1–3]. In particular, a lot of effort has been devoted to the synchronization and control of complex dynamical networks [4–9]. It has been shown that the states of all nodes in a connected network of chaotic oscillators with a suitable coupling strength will synchronize and the synchronized state may still be chaotic [1,2]. However, by applying local feedback control to a small fraction of network nodes, it is possible to stabilize a dynamical network at a homogeneous equilibrium point [7,8]. This implies that the state of every node in the network converges to an equilibrium point of an isolated node. Such a stabilization goal can even be achieved by just controlling one node randomly selected from the network [9].

However, many technological, social and biological networks divide naturally into groups, and nodes in the same group often have the same kind of function [10–16]. For example, when a crowd of people choose to accept or oppose an opinion according to their preferences, two groups are often formed: one group is made up of those who accept the opinion; the other one is made up of those who oppose the opinion [10]. Communities in neuronal networks and biochemical networks may also represent functional groups [12–14] and pages on relevant topics may make up groups on the Web [15]. This motivates the following question: Is there a distributed control algorithm such that states of nodes in different groups stabilize on different goals?

In this work, the problem of controlling a complex dynamical network to attain an inhomogeneous equilibrium point is investigated; this means that nodes in the same group achieve the same equilibrium point as an isolated node, while different groups correspond to different points. In this case, the difficulty is that the desired goal is usually not an equilibrium point of the whole network. To deal with this difficulty, an open-loop constant control approach is first proposed for making the desired goal an inhomogeneous equilibrium point of the network. Then the feedback pinning control approach is applied to make the inhomogeneous equilibrium point asymptotically stable.

The paper is organized as follows. In Section 2, the preliminaries are introduced. An open-loop constant control and feedback pinning control are investigated in Sections 3 and 4, respectively. In Section 5, simulation results are proposed. In the last section, conclusions are presented.

2. Preliminaries

Consider a network of N diffusively linearly coupled identical oscillators, with each oscillator being an n-dimensional dynamical

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system. The state equations of the network are

$$\dot{x}_i(t) = f(x_i(t), t) + c \sum_{j=1, j\neq i}^{N} a_{ij}(x_j(t) - x_i(t)) + u_i(t),$$
$$i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in R^n$ is the state variable of node i, f describes the dynamics of each isolated node, the constant $c > 0$ represents the coupling strength. $A = (a_{ij})_{N\times N}$ is the coupling matrix which is defined as follows: If there is a connection between node i and node j, then $a_{ij} = a_{ji} > 0$ $(i \neq j)$; otherwise, $a_{ij} = a_{ji} = 0$ $(i \neq j)$. $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$ $(i = 1, 2, \ldots, N)$. $u_i(t)$ is the control input added at node i. Synchronization of network (1) without control (i.e., $u_i(t) \equiv 0$) has been widely investigated [7–9,17].

Suppose that the network is connected in the sense that there are no isolated clusters in the network. This implies that the coupling matrix A is symmetric and irreducible, with eigenvalues that can be ordered as $0 = \lambda_1(A) > \lambda_2(A) \geq \cdots \geq \lambda_N(A)$ [5,6].

The dynamics of an isolated node without control can be written as

$$\dot{y} = f(y). \tag{2}$$

Each isolated node is assumed to have p equilibrium points $x_{ei} \in R^n$ $(i = 1, 2, \ldots, p)$, which means $f(x_{ei}) = 0$ $(i = 1, 2, \ldots, p)$, where $p > 1$. Suppose that each node in network (1) has to be stabilized at one of the p equilibrium points of an isolated node using distributed control. Without loss of generality, the nodes are divided into p clusters; the sets of subscripts of these clusters are $G_1 = \{1, 2, \ldots, N_1\}$, $G_2 = \{N_1 + 1, N_1 + 2, \ldots, N_1 + N_2\}, \ldots, G_p = \{N_1 + \cdots + N_{p-1} + 1, \ldots, N\}$, where $N_1 + N_2 + \cdots + N_p = N$. The nodes in the cluster G_i $(i = 1, 2, \ldots, q)$ have to be stabilized on x_{ei}, where $1 \leq q \leq p$. The problem considered here is that of designing distributed control inputs $u_i(t)$ $(i = 1, 2, \ldots, N)$ so that as $t \to +\infty$, we have

$$\begin{cases} x_1(t) = x_2(t) = \cdots = x_{N_1}(t) = x_{e1} \\ x_{N_1+1}(t) = x_{N_1+2}(t) = \cdots = x_{N_1+N_2}(t) = x_{e2} \\ \vdots \\ x_{N_1+\cdots+N_{q-1}+1}(t) = x_{N_1+\cdots+N_{q-1}+2}(t) = \cdots = x_N(t) = x_{eq}. \end{cases} \tag{3}$$

In the following, an open-loop constant control approach is first proposed for making the point defined by (3) an inhomogeneous equilibrium point of the controlled network (1); then the feedback pinning control approach is applied to make the inhomogeneous equilibrium point asymptotically stable.

3. Open-loop constant control

The following constant control input is considered:

$$u_i(t) = -c \sum_{j=1, j\neq i}^{N} a_{ij}(s_i - s_j), \quad i = 1, 2, \ldots, N. \tag{4}$$

Here s_i is the desired state of node i, i.e.,

$$\begin{cases} s_1 = s_2 = \cdots = s_{N_1} = x_{e1} \\ s_{N_1+1} = s_{N_1+2} = \cdots = s_{N_1+N_2} = x_{e2} \\ \vdots \\ s_{N_1+\cdots+N_{q-1}+1} = s_{N_1+\cdots+N_{q-1}+2} = \cdots = s_N = x_{eq}. \end{cases} \tag{5}$$

Note that if node i does not connect to any nodes in other groups, then $u_i(t) = 0$. It is easy to verify that the desired state defined in (3) is an inhomogeneous equilibrium point of the network (1) with controller (4). Clearly, such an inhomogeneous equilibrium point may not be stable. However, the following numerical example shows that each node's state will be close to its desired state, so the nodes in different groups can still be distinguished easily.

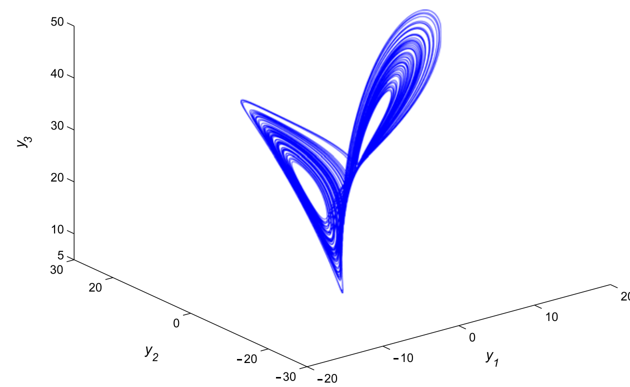

Fig. 1. Chaotic behavior of the Lorenz oscillator.

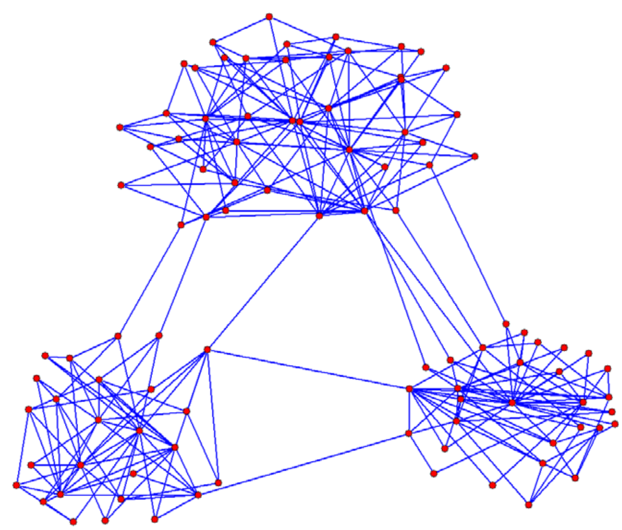

Fig. 2. A community network model of $N = 100$ nodes with $q = 3$ communities.

Suppose that each isolated node is a Lorenz oscillator described by

$$\begin{cases} \dot{y}_1 = -10y_1 + 10y_2 \\ \dot{y}_2 = 28y_1 - y_2 - y_1y_3 \\ \dot{y}_3 = y_1y_2 - 2.67y_3. \end{cases} \tag{6}$$

The Lorenz oscillator has a double-scroll chaotic attractor, as shown in Fig. 1 [18]. $[8.4853, 8.4853, 27]^T$, $[-8.4853, -8.4853, 27]^T$ and $[0, 0, 0]^T$ are three equilibrium points of the Lorenz oscillator.

A community network is constructed by using the algorithm in [16]. The network has 100 nodes and the sizes of the three communities are 28, 34 and 38 (Fig. 2). Obviously, the connections existing in the same community are much denser than the connections between different communities. Suppose that the desired states of nodes in three communities are three equilibrium points of the Lorenz oscillator. A constant controller of the form (4) is added at each node and we set the coupling strength $c = 50$. Starting from random initial states, the states of nodes in the same community will be close to but not exactly equal to their desired states (Fig. 3).