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Return map structure and entrainment in a time-state-scale re-entrant system

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Abstract

From the perspective of a heterarchy, endo-physics, or internal measurement, a time-state-scale re-entrant system has been proposed [Y.-P. Gunji, K. Sasai, S. Wakisaka, BioSystems (submitted for publication)]. However, the dynamical structure of this system has yet to be estimated. Because the return map of the time-space re-entrant system results from the twisted coupling between the temporal and state-scale states, it is analytically determined. The attractors of entrainment in the interactive re-entrant system are also determined by a particular recursive rule, and it is also shown that a resulting return map can control the attractors of the interactive systems.

Keywords: Self-reference; Chaotic liar; Frame problem; Subjective time

1. Introduction

We formalized self-reference to reduce the occurrence of contradictions in the time-state-scale re-entrant system by connecting the notions of self-reference and the frame problem [1]. In establishing self-reference, one must indicate the reference of wholeness, which is possible only in a restricted (or privately closed) context. The frame problem is nothing but a statement pointing out the impossibility of indicating wholeness in advance. Therefore, the frame problem can invalidate the hidden assumption of self-reference.

The concept of self-reference is often replaced with a dynamical system, although the frame problem is not positively expressed. We replace self-reference with the temporal rule and the frame problem with the state-scale rule, and define a twisted coupling between them. It has been reported that the time-state-scale re-entrant system is a self-similar map defined in terms of a return map. In this paper, we show that a self-similar return map dependent on a series of states can be expressed as a strict map, and the structure plays a role in the pattern of entrainment in the interactive re-entrant system.

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2. Interaction between Gasket Maps

2.1. Definition of TSSRS

First, we review the time-state-scale re-entrant system (TSSRS) [1]. Given a logical expression such as $z = f(z)$ with a variable z, one can sometimes find a contradiction. If z is either 0 or 1 and $f(z) = 1 - z$, equality does not hold. Since an expression contains a whole z as a part in the form of $f(z)$, it is called a self-referential form. To remove a contradiction, one can introduce time-shift, such as $z^{t+1} = f(z^t)$. Especially in [2], a self-referential form containing a contradiction is reduced to a dynamics called the chaotic liar, e.g. $z^{t+1} = f(z^t) = 1 - |2z^t - 1|$, where z^t is a real value in [0.0, 1.0]. Despite introducing a time-shift, a problem remains with regards to the origin of initial state z^0. The initial value problem is embedded in a computational process if a real value is approximated by a finite binary sequence, since the smallest digit is influenced by smaller digits that are discarded. In this sense, the initial value problem is replaced by a transition rule making a larger digit from smaller hidden digits. We call such a transition rule the state-scale shift, and denote it g.

Finally, two problems arise from the logical expression $z = f(z)$. One results from introducing a time-shift such as $z^{t+1} = f(z^t)$ and concerns the point from where the initial value is derived. The other results from introducing the state-scale shift, g, and concerns the nature of the smaller hidden digits. If one takes two problems separately, neither can be resolved. The novel concept presented by the time-state-scale re-entrant form is to handle both of them simultaneously. An initial value is supplied by a state-scale shift, and hidden digits are supplied by a time-shift. That is, $z^{t+1} = f(\text{twist}\,1(x^t))$ and $x^{t+1} = g(\text{twist}\,2(z^t))$, where z^t and x^t are binary sequences and twist 1 and twist 2 are particular maps controlling the updating of binary sequences. As a result, the time-state-scale re-entrant form develops a binary sequence from negotiating the temporal shift and state-scale shift.

The time-state-scale re-entrant form consists of the temporal-rule, time-state z^t in [0.0, 1.0], the state-scale rule, and state-scale state x^t in [0.0, 1.0] [1]. Both rules are derived from one self-referential form called the chaotic liar [2]. The development of the re-entrant form is executed as a calculation of binary sequences. The state-scale state designated by x^t is expressed as a binary sequence such as

$$x^t = \sum_{i=0}^{N-1} 2^{-(i+1)} a_s^t(i), \tag{1}$$

with $a_s^t(i) \in \{0, 1\}$, $s = 0, 1, \ldots, N$. The state-scale transition of x^t is determined by

$$\begin{aligned} a_{s+1}^t(i) &= a_s^t(i+1) \quad \text{if } a_s^t(0) = 0; \\ &1 - a_s^t(i+1) \quad \text{otherwise}, \end{aligned} \tag{2}$$

that is, the state-scale rule. The temporal state is obtained via y^t expressed as

$$y^t = \sum_{s=1}^{N} 2^{-(N+1-s)} a_s^t(0). \tag{3}$$

From the state of y^t, the temporal-state, z^t, is obtained by

$$\begin{aligned} z^t &= y^t/(1-y^t) \quad \text{if } y^t < (1-y^t); \\ &(1-y^t)/y^t \quad \text{otherwise}, \end{aligned} \tag{4}$$

where the calculation is executed in a binary fashion, and z^t is expressed as

$$z^t = \sum_{k=0}^{N-1} 2^{-(k+1)} b^t(k), \tag{5}$$

with $b^t(k) \in \{0, 1\}$. The temporal state is provided for the next state-scale state by

$$a_0^{t+1}(k) = b^t(N-1-k), \tag{6}$$

with $k = 0, 1, \ldots, N-1$. The calculation procedure is schematically shown in Fig. 1.

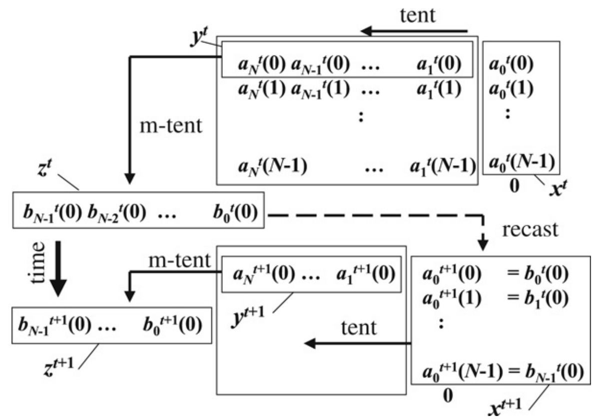

Fig. 1. Schematic diagram for the procedure of computing the time-space re-entrant system. Given a boundary condition (x^t) for N-length binary sequence, as shown vertically at the t-th step, a tent map (Eq. (2)) is applied and iterated N times. As a result, a new sequence (y^t), as shown horizontally, is obtained. A modified tent map (Eq. (4)) is then applied, and the resulting sequence (z^t) recasts a boundary condition at the $(t+1)$-th step. A procedure identical to that of the t-th step leads to a binary sequence of z^{t+1}, creating progress in terms of time.

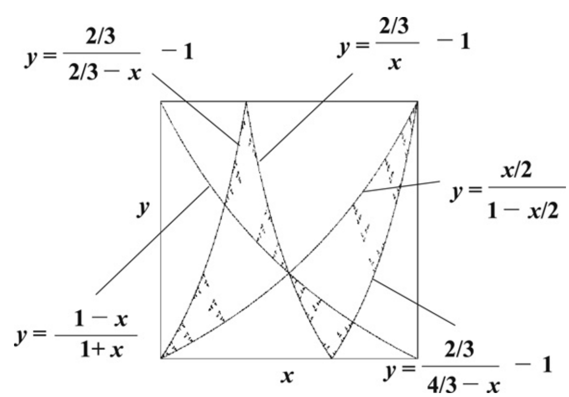

Fig. 2. A return map resulting from computation of the time-space re-entrant system, in terms of z^{t+1} plotted against z^t. This is termed a gasket map. The upper and lower margins are determined in the form of functions as shown in the graph, where z^{t+1} and z^t are represented by y and x, respectively.

2.2. Analysis of dynamical structure in TSSRS

As shown previously, the return map of (z^t, z^{t+1}) shows a self-similar gasket pattern [1]. The function of the upper and lower margins of the gasket can be analytically determined (Fig. 2). In order to reduce the functions, we first investigate the map of (z^t, y^{t+1}), and then reduce the map of (z^t, z^{t+1}). As shown in Fig. 3, the upper and lower margins of y^{t+1} against z^t can be expressed as

$$1.5z^t \geqq y^{t+1} \geqq 0.5z^t. \tag{7}$$

The above can be proven by induction with respect to the length of a binary sequence.

Consider the case in which the length of a binary sequence x^t is 1. Therefore, $N = 1$ and $b^t(0)$ is either 0 or 1. Via Eq. (6), $a_0^{t+1}(0) = b^t(0)$. The next state, y^{t+1}, is calculated as,

$$\begin{array}{lccc} a_1^{t+1}(0)\ a_0^{t+1}(0) & \boxed{0}\ \boxed{0} & \boxed{1}\ \boxed{1} \\ \phantom{a_1^{t+1}(0)}\ a_0^{t+1}(1) & 0 & 0. \end{array} \tag{8}$$

Two cases such that $a_0^{t+1}(0) = 0$ and 1 are shown, and in these cases the left and right boxes represent binary expressions of y^{t+1} and z^t, respectively. Because $a_0^{t+1}(1)$ is defined as blank, it is regarded as having a value of 0. Thus, if