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Dynamic and static excitations of a classical discrete anisotropic Heisenberg ferromagnetic spin chain

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Abstract

Using Jacobi elliptic function addition formulas and summation identities we obtain several static and moving periodic soliton solutions of a classical anisotropic, discrete Heisenberg spin chain with and without an external magnetic field. We predict the dispersion relations of these nonlinear excitations and contrast them with that of magnons and relate these findings to the materials realized by a discrete spin chain. As limiting cases, we discuss different forms of domain wall structures and their properties.

Keywords: Anisotropic Heisenberg ferromagnetic chain; Spin chain; Nonlinear excitations; Solitons; Peierls–Nabarro barrier

1. Introduction

Establishing integrability and obtaining exact solutions of discrete nonlinear physical systems are important issues of current interest. Starting with the integrable discrete model of Ablowitz and Ladik [1], for several other discrete nonlinear evolution equations exact elliptic function and soliton solutions have been obtained in recent years [2]. These include certain discrete versions of the nonlinear Schrödinger (NLS) equation [3,4], ϕ^4 equation [5,6], derivative NLS equation [7], coupled ϕ^4 equation [8], coupled asymmetric double well and coupled ϕ^6 equation [9], complex modified Korteweg–de Vries equation [10], etc., where effective use of summation relations of Jacobian elliptic functions was made and periodic and solitary wave solutions of moving and static types were obtained.

In this connection, a physically important discrete nonlinear dynamical system which has been of considerable interest in diverse areas of physics for a long time is the anisotropic Heisenberg ferromagnetic (and antiferromagnetic) spin system with or without an external magnetic field. It has been studied for various aspects in magnetism, condensed matter physics/materials science, statistical physics, nonlinear dynamics, etc. both from classical and quantum points of view [11]. For example, the one-dimensional quantum spin-$1/2$ *XYZ* chain has been shown to be an exactly solvable system either through the Bethe ansatz procedure or through the quantum inverse scattering method [12,13] and the eigenvalue spectrum and eigenfunctions have been obtained. For large values of spins, however, a classical/quasi-classical description has been known to be an adequate description so that spins can be treated as unit vectors and classical equations of motion for the spin vectors can be obtained as limiting forms of the quantum equation of motion or as dynamical equations derived from postulated spin Poisson bracket relations [14]. Another area of considerable physical interest in which such classical anisotropic spin systems have been studied in the presence of Gilbert damping is the microscopic behavior of spin waves in magnetic bodies of arbitrary shape [15] and the study of spin-torque effect in ferromagnetic layers with spin currents [16] on spin waves and domain walls. Recently it has also been pointed out that discrete breathers can exist in anisotropic spin chains with additional onsite anisotropy [17]. In any case, the resultant

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equations of motion describe an extremely interesting class of discrete nonlinear dynamical systems and exploration of the underlying dynamical properties is of interest both from the theoretical and applied physics points of view.

The one-dimensional Heisenberg ferromagnetic spin system with nearest neighbor exchange interaction has been shown to possess several completely integrable soliton bearing systems in its *continuum limit*: (i) the pure isotropic case [18,19], (ii) the uniaxial anisotropic case [20], and (iii) the biaxial anisotropic case [21]. These systems also have a strong connection with the nonlinear Schrödinger equation [22]. However, till date no exactly integrable *discrete* dynamical Heisenberg spin system has been identified in the literature, although a variant of the system, namely the Ishimori spin chain, is known to be completely integrable [23]. It is generally expected that the discrete anisotropic Heisenberg spin chain is a nonintegrable nonlinear dynamical system. Yet, as we show in this article, a number of interesting exact periodic and stationary structures, including domain wall type structures, for the fully anisotropic system (*XYZ* case as well as the limiting *XYY* and planar *XY* cases) can be obtained and their properties analyzed using standard techniques. In fact, Roberts and Thompson [24] and Granovskii and Zhedanov in a series of papers [25–27], have obtained special classes of solutions for the anisotropic spin system. In particular, the latter authors have shown that the time-independent case of the *XYZ* anisotropic spin chain is an integrable map by relating it to a Neumann type discrete system [26], see also Ref. [28].

In this paper, by parametrizing the unit spin vector in terms of the basic Lamé polynomials of lower order [29] or their derivatives and by a judicious use of various addition theorems and summation relations obeyed by Jacobi elliptic functions [30] we point out that several classes of explicit dynamical and static structures can be obtained. In the limiting cases we obtain linear spin wave solutions and different nonlinear domain wall type solutions in a natural way. We study the physical implications of these solutions like the energy spectrum, effect of discreteness such as the Peierls–Nabarro barrier [31–33], linear stability and so on.

The plan of the paper is as follows. In Section 2, we introduce the dynamical equations of motion and introduce certain natural parametrizations of the unit spin vector. In Section 3, we obtain two classes of periodic solutions, investigate the associated dispersion relations and energy expressions and indicate a semiclassical quantization of these solutions. In Section 4 we report various classes of static solutions for the *XYZ*, *XYY* and *XY* planar models. In Section 5, we obtain the total energy expressions associated with the various static solutions and discuss the effect of discreteness including the Peierls–Nabarro potential barrier. In Section 6, the isotropic case is considered, while in Section 7 the linear stability of both time periodic and static solutions is investigated. Then in Section 8, we present some explicit time-dependent solutions for the case when the onsite anisotropy or an external magnetic field is introduced. Finally, in Section 9 we summarize our results. In Appendix A we include some of the relevant addition theorems and summation relations obeyed by the Jacobi elliptic functions required for our analysis, while in Appendix B some details on semiclassical quantization are given.

2. The Heisenberg anisotropic spin chain

2.1. Equation of motion

We consider a one-dimensional anisotropic Heisenberg ferromagnetic spin chain with the spin components $\vec{S}_n = (S_n^x, S_n^y, S_n^z)$, satisfying the constraint of unit length

$$(S_n^x)^2 + (S_n^y)^2 + (S_n^z)^2 = 1, \tag{1}$$

modeled by the Hamiltonian

$$H = -\sum_{\{n\}}(AS_n^x S_{n+1}^x + BS_n^y S_{n+1}^y + CS_n^z S_{n+1}^z) - D\sum_n (S_n^z)^2 - \vec{\mathcal{H}} \cdot \sum_n \vec{S}_n, \tag{2}$$

where the sum is over the nearest neighbors, A, B and C are the (exchange) anisotropy parameters, D is the onsite anisotropy parameter and $\vec{\mathcal{H}} = (\mathcal{H}, 0, 0)$ is the external magnetic field along the x direction (for convenience). For the *XYZ* model, $A \neq B \neq C$, $D = 0$ and for the *XY* model $C = 0$ and $D = 0$. Using the spin Poisson bracket relation [14]

$$\{S_i^\alpha, S_j^\beta\}_{PB} = \delta_{ij}\epsilon_{\alpha\beta\gamma} S_j^\gamma, \quad \alpha,\ \beta,\ \gamma = 1, 2, 3, \tag{3}$$

where δ_{ij} is the Kronecker delta and $\epsilon_{\alpha\beta\gamma}$ is the Levi-Civita tensor, for any two functions $\mathcal{A}$ and $\mathcal{B}$ of spins one has

$$\{\mathcal{A}, \mathcal{B}\}_{PB} = \sum_{\alpha,\beta,\gamma}\sum_{i=1}^{N} \epsilon_{\alpha\beta\gamma} \frac{\partial \mathcal{A}}{\partial S_i^\alpha} \frac{\partial \mathcal{B}}{\partial S_i^\beta} S_i^\gamma, \tag{4}$$

and the equation of motion becomes

$$\frac{\mathrm{d}\vec{S}_n}{\mathrm{d}t} = \vec{S}_n \times [A(S_{n+1}^x + S_{n-1}^x)\vec{i} + B(S_{n+1}^y + S_{n-1}^y)\vec{j} + C(S_{n+1}^z + S_{n-1}^z)\vec{k} + 2DS_n^z\vec{k}] + \vec{S}_n \times \vec{\mathcal{H}}, \quad n = 1, 2, \ldots, N, \tag{5}$$

where $\vec{i}$, $\vec{j}$, $\vec{k}$ form a triad of Cartesian unit vectors. Explicitly, in component form the above equation reads

$$\frac{\mathrm{d}S_n^x}{\mathrm{d}t} = CS_n^y(S_{n+1}^z + S_{n-1}^z) - BS_n^z(S_{n+1}^y + S_{n-1}^y) - 2DS_n^y S_n^z, \tag{6}$$

$$\frac{\mathrm{d}S_n^y}{\mathrm{d}t} = AS_n^z(S_{n+1}^x + S_{n-1}^x) - CS_n^x(S_{n+1}^z + S_{n-1}^z) + 2DS_n^x S_n^z + \mathcal{H}S_n^z, \tag{7}$$

$$\frac{\mathrm{d}S_n^z}{\mathrm{d}t} = BS_n^x(S_{n+1}^y + S_{n-1}^y) - AS_n^y(S_{n+1}^x + S_{n-1}^x) - \mathcal{H}S_n^y. \tag{8}$$

Eq. (5) or (6)–(8) can also be obtained as the limiting case of the corresponding quantum dynamical equation of motion for the spin operators when $\hbar \to 0$ or $S \to \infty$. In either case, the dynamics are obtained by solving the initial value problem