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Coupled map lattices as spatio-temporal fitness functions: Landscape measures and evolutionary optimization

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Abstract

Coupled Map Lattices (CML) can be interpreted as spatio-temporal fitness landscapes which may pose a dynamic optimization problem. In this paper, we analyze such dynamic fitness landscapes in terms of the landscape measures modality, ruggedness, information content and epistasis. These measures account for different aspects of problem hardness. We use an evolutionary algorithm to solve the dynamic optimization problem and study the relationship between performance criteria of the algorithm and the landscape measures. In this way we relate problem hardness to expectable performance.

Keywords: Coupled map lattices; Fitness landscapes; Optimization; Evolutionary algorithm

1. Introduction

The concept of fitness landscapes originates from evolutionary biology and is an important tool in theoretical studies in evolutionary optimization [1–4]. Such a fitness landscape assigns fitness values to the points of a search space in which an optimal solution is to be found. This search space can be constructed by a genotype-to-fitness mapping or more generally by encoding the set of possible solutions of an optimization problem to span a representation space for which additionally a neighborhood structure needs to be defined. It hence can be viewed as a more or less mountainous landscape that consists of hills and valleys, peaks and ridges, plains and plateaus. Optimization in such a landscape means detecting the coordinates and the magnitude of the highest hill (or lowest valley). Part of the usefulness of the fitness landscape metaphor stems from this visual image as it provides intuitively geometrical concepts of what an optimization problem is and what kind of difficulties can be expected in solving it. For analyzing and quantifying fitness landscapes, so-called landscape measures have been proposed.

Typically, the topology of the fitness landscape is considered not to be changing over the run-time of the evolutionary algorithm and hence such a fitness landscape is a static concept. As dynamic optimization became more and more an important topic in evolutionary computation [5–8], concepts of fitness landscapes in dynamic environments were needed in turn. Whereas recent results have shown that such an approach is useful for characterizing and classifying types of dynamic landscapes [9], there is a certain lack of environments that show sufficiently complex structure in both spatial topology and temporal dynamics.

A general spatio-temporal fitness landscape may describe changes of the fitness values continuously in both space and time and could be modelled by a PDE. The idea of employing CML as a fitness landscape comes from the intention to use a discretization of space and time in order to facilitate efficient computing of the fitness landscape. In addition, CML have been the subject of intensive research, which revealed a broad variety of complex spatio-temporal behavior, including different types of pattern formation, spatio-temporal chaos, quasi-periodicity and emergence; see e.g. [10,11] and references therein.

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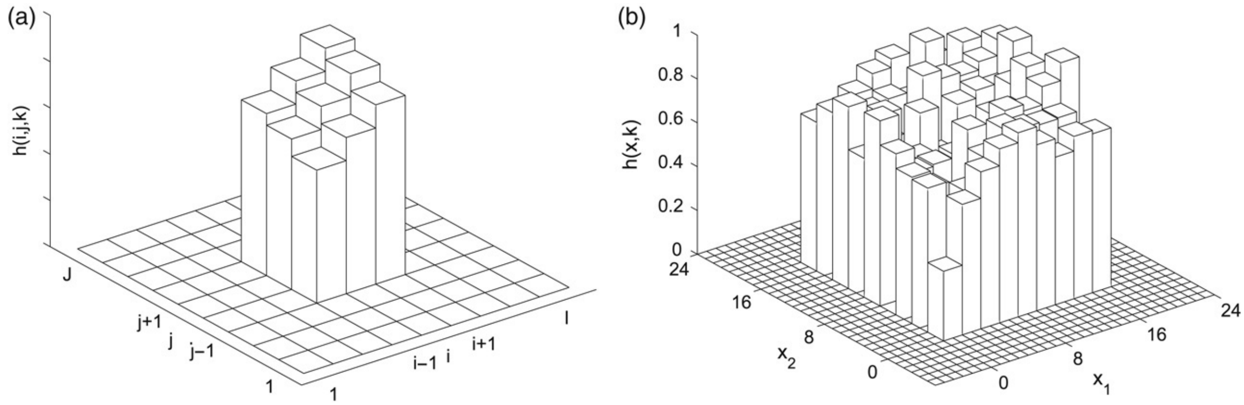

Fig. 1. Coupled map lattices: (a) Generic structure. (b) Spatio-temporal fitness landscape for $I = J = 9$ and $s_1 = s_2 = 0.5$.

In a recent work [12], spatio-temporal fitness landscapes were introduced that are constructed from spatio-temporal dynamical systems, namely from CML. This paper extends and generalizes these results. In the next section, a methodology for constructing spatio-temporal fitness landscapes from CML is given. In Section 3, these landscapes are analyzed in terms of landscape measures; these are modality, ruggedness, information content and epistasis. In addition, the relationships between these landscape measures are studied. In Section 4, factors contributing to problem hardness in dynamic optimization such as change frequency and dynamic severity are reviewed. Numerical experiments using an evolutionary algorithm to optimize in the spatio-temporal fitness landscape are reported in Section 5. These results are interpreted in terms of performance criteria for optimization. We link these performance data with the landscape measures in order to relate measures to expectable performance. In the final section, the findings are summarized and conclusions are drawn.

2. Constructing spatio-temporal fitness landscapes from CML

The section is concerned with constructing spatio-temporal fitness landscapes and this should begin with defining a spatio-temporal dynamical system. A general spatio-temporal fitness landscape may describe the evolution of fitness values in a search space where changes may occur continuously in both space and time. Such a spatio-temporal evolution may be modelled by a PDE. To facilitate efficient computing, an appropriate discretization is needed, the more so as numerical effort in solving the dynamic optimization problem by an evolutionary algorithm scales with the number of fitness function evaluations. CML have been shown to constitute such a discretization of space and time, in particular for reaction–diffusion systems, where important features of the dynamics are preserved, e.g. [10,13]. For a link between reaction–diffusion dynamics and fitness landscapes, see [14].

As a first step in defining spatio-temporal fitness landscapes from CML, we lay out a lattice grid with $I \times J$ equally sized cells, which form a 2D structure. For every discrete time step k, $k = 0, 1, 2, \ldots$, each cell is characterized by its height

$$h(i, j, k), \quad i = 1, 2, \ldots, I, \; j = 1, 2, \ldots, J, \tag{1}$$

where (i, j) denote the spatial indices in vertical and horizontal directions, respectively; see Fig. 1. We interpret this height $h(i, j, k)$ as fitness according to the geometrical metaphor of a fitness landscape. It is subject to changes over time, which we shall describe here using two-dimensional CML with nearest-neighbor coupled interaction [10,11]

$$\begin{aligned} h(i, j, k+1) = {} & (1-\epsilon) g(h(i, j, k)) + \frac{\epsilon}{4} \left[g\left(h(i-1, j, k)\right) \right. \\ & + g\left(h(i+1, j, k)\right) + g\left(h(i, j-1, k)\right) \\ & \left. + g\left(h(i, j+1, k)\right) \right], \end{aligned} \tag{2}$$

where $g(h(i, j, k))$ is a local mapping function and ϵ is the diffusion coupling strength. In other words, as $h(i, j, k)$ denotes the height of the $h(i, j)$-th unit bar situated in the (i, j)-lattice cell at time step k, Eq. (2) describes how this height changes over time depending on its own height and the heights of the surrounding bars; see Fig. 1(a).

To complete the definition of the CML (2), we employ the logistic map

$$g(h(i, j, k)) = \alpha h(i, j, k)(1 - h(i, j, k)) \tag{3}$$

as mapping function and define the periodic boundary conditions

$$\begin{aligned} h(I+1, j, k) &= h(1, j, k), \\ h(i, J+1, k) &= h(i, 1, k). \end{aligned} \tag{4}$$

Initialization of the CML is done with initial heights $h(i, j, 0)$ being realizations of a random variable uniformly distributed on $[0, 1]$.

The CML are spatio-temporal dynamical systems with discrete space (lattice) and time (map). The system's states, which we interpret as heights, are continuous and situated on the lattice. They dynamically interact with surrounding states via the nonlinear map (3). As a result of this interaction, the CML are known to exhibit a rich spatio-temporal behavior, including different types of spatio-temporal periodicity and chaos, quasi-periodicity and pattern formation. The spatio-temporal behavior of the CML depends on the lattice size $I \times J$ and two parameters, the coupling strength ϵ and the nonlinear parameter α. These parameters span a parameter space in which different regions represent different types of spatio-temporal behavior.