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Opinion spreading and agent segregation on evolving networks

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Abstract

We study a stochastic model where the distribution of opinions in a population of agents coevolves with their interaction network. Interaction between agents is enhanced or penalized according to whether they succeed at reaching an agreement or not. The system evolves towards a state where the network's structure and the opinion distribution is frozen, and the population is divided into disconnected communities. The structural properties of the population in the final state vary considerably with the control parameters. By means of numerical simulations, we give a detailed account of such properties, as well as of the final opinion distribution. We also provide approximate analytical results which explain some of the numerical results and clarify their origin.

Keywords: Self-organization; Network evolution; Social systems

1. Introduction

In the last one or two decades, virtually hundreds of papers in the physics literature have introduced and analyzed models involving coupled dynamical elements whose interaction pattern is represented by a network [1,2]. While some of them aim at producing generic knowledge about collective behaviour in complex systems, and are thus formulated in rather abstract terms, others are designed so as to closely represent the dynamics of specific real systems, of interest in basic and applied sciences. In these models, both the coupled dynamical elements, or agents, and their interaction network may evolve with their own rules. Among other goals, they have been used to explain the emergence of non-trivial properties in the structure of networks, such as power-law connectivity distributions and high clustering in otherwise random graphs, and coherent spatiotemporal dynamics, such as spontaneous synchronization and pattern formation.

Regarding the time scales involved in the dynamical rules of agents and networks, most of the models referred to above can be divided into two broad classes. In the first class, the dynamics of agents is much faster than that of their interaction network. The extreme examples in this class are those models where the agent population evolves on a quenched network [3]. Other models in this class admit that the network changes slowly, responding to some "selection pressure" determined by the agents' collective state [4]. The second class, on the other hand, includes those models where the network evolves much faster than the agents. In the extreme cases, as in models for network growth [5], there is no agent dynamics at all. The intermediate regime, where the two time scales are similar and one can thus speak about coevolution of agents and network, has been relatively much less studied (see, however, Ref. [6]).

The coevolution of agents and their interaction network is essential to the collective dynamics of social systems, in particular, to explain the ubiquitous occurrence of segregation. In human societies, for instance, it is typical that many communities with mutually excluding cultural traits – which may involve religious beliefs, professional or generational jargons, artistic inclinations, etc. – coexist in the same geographical region [7,8]. With respect to those traits, their interaction is infrequent, so they can be considered as effectively segregated from each other. A key mechanism in this kind of social segregation is the feedback between the construction of agreement within a community and the enhancement of distinctions with other communities: specific traits become better established as differences between communities develop and grow [9].

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In this paper, we analyze a recently introduced model for the coevolution of a population of agents and their interaction network [10]. The model describes the spreading of a binary opinion [11]. Interactions between agents with similar or different opinions are respectively favoured or penalized. It has already been shown that, in spite of the simple evolution rules, the population can reach a variety of social patterns. Here, we give a more detailed description of the statistical properties of those different patterns, and extend the analytical treatment advanced in [10] to explain some of the numerical results.

2. The model

The system consists of a population of N agents, situated at the nodes of a network. The state of each agent i is given by a binary variable m_i, which adopts the values $+1$ or -1. It represents the agent's opinion. A pair of agents is said to be connected if a network link joins the corresponding nodes. At each step, a pair of connected agents is chosen at random from the whole population. If the two agents have the same opinion, nothing happens. Otherwise, with probability p_1 either agent adopts the other agent's opinion, so that the two opinions become identical. With the complementary probability $1 - p_1$, opinions are not changed. In this case, however, the link between both agents is removed with probability p_2, and the interaction network loses one of its links. Note that, for $p_2 = 0$, these dynamical rules coincide with those of the voter model, which has been extensively studied both in regular lattices [12] as well as in complex networks [13].

Under the action of the above dynamics, the system will eventually reach a frozen, absorbing state where, generally, the population is split into disconnected communities. Within a given community, all agents share the same opinion. This final state depends on the initial condition and on the specific realization of the stochastic evolution. In our numerical simulations, we choose an initial condition where the population is fully connected so that, at that stage, any pair of agents can potentially interact. With respect to the segregation process, this is in a sense the less favourable initial state. As for the agents' opinions, they are initially distributed with equal probability over the population.

The probabilities p_1 and p_2 determine the frequencies of the two processes that drive the dynamics — opinion flips and link removal. With respect to the evolution of the system, these probabilities are independent control parameters. The statistical properties of the final state, however, are completely determined by the relative frequencies of the events that effectively change the state of the system. In other words, they depend on p_1 and p_2 through a certain combination only. To quantify this feature, let us call $p_-(t)$ the fraction of links connecting agents with different opinions. The probability that any agent changes its opinion at a given step is

$$\pi_1(t) = p_-(t) p_1, \tag{1}$$

while the probability that a connection is deleted is

$$\pi_2(t) = p_-(t)(1 - p_1) p_2. \tag{2}$$

The sum $\pi(t) = \pi_1(t) + \pi_2(t)$ is the probability per evolution step that any change takes place, and thus fixes an overall evolution time scale. If p_1 and p_2 vary in such a way that the ratios $q = \pi_1(t)/\pi(t)$ and $1 - q = \pi_2(t)/\pi(t)$ are kept constant, such an overall time scale will change, but the relative frequency of the two processes will be the same, leading to statistically equivalent final states. Thus, the only independent combination of the probabilities p_1 and p_2 relevant to the determination of the final state is

$$q = \frac{\pi_1(t)}{\pi(t)} = \frac{p_1}{p_1 + (1 - p_1) p_2}. \tag{3}$$

Since our discussion will hereafter focus on the final structure of the population and the corresponding distribution of opinions, q will play the role of the only control parameter in the model (besides the population size N). As defined in Eq. (3), this parameter is essentially a renormalization of the opinion-flip probability p_1.

At the two extreme values of q, the final state of the population is immediately inferred from the evolution rules. For $q = 0$ (i.e. $p_1 = 0$, provided that $p_2 \neq 0$), no opinion flips can take place. Therefore, the population splits into two mutually disconnected communities with similar sizes and opposite opinions. Internally, each community stays fully connected, so that the total number of remaining connected pairs is close to $2(N/2)(N/2 - 1)/2 \approx N^2/4$. As we show in Section 4.1, the number of steps needed to reach the final state for $p_1 = 0$ is of order N^2/p_2.

For $q = 1$ (i.e. $p_1 = 1$), on the other hand, interacting agents with initially opposite opinions always reach consensus, so that no interaction links are deleted. At the final state, all agents share the same opinion and the network is still fully connected, with its $N(N - 1)/2 \approx N^2/2$ links intact. As discussed in detail in Section 4.1, the number of agents with each opinion performs a random walk, until it reaches either N or zero. The whole process, thus, is driven by fluctuations. The total number of steps is of order N^2 and, naturally, does not depend on the probability of link removal p_2.

3. Numerical results

3.1. Mean opinion and the fraction of remaining links

The two limits of the control parameter q discussed in the last section suggest that a first numerical quantification of the opinion distribution and the network structure in the final state is given, respectively, by the mean opinion

$$m = \frac{1}{N} \sum_{i=1}^{N} m_i, \tag{4}$$

and the fraction of remaining links

$$r = \frac{2P}{N(N - 1)}, \tag{5}$$

where P is the final number of connected pairs of agents.