Physica D 291 (2015) 1–7

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Stability of equilibrium state in a laser with rapidly oscillating delay feedback

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HIGHLIGHTS

- We study stability of lasing under periodically modulated time delay feedback.
- Minimum level of destabilizing FB versus amplitude modulation is found.
- Control of stationary lasing fails in vicinity of subcritical bifurcation boundary.
- We describe analytically limit cycle whose amplitude depends on modulation frequency.

ARTICLE INFO

Article history:
Received 19 February 2014
Received in revised form
5 July 2014
Accepted 3 October 2014
Available online 13 October 2014
Communicated by V.M. Perez-Garcia

Keywords:
Laser dynamics
Optical feedback
Bifurcation analysis

ABSTRACT

Dynamics of laser with time-variable delayed feedback is analyzed in the neighborhood of the equilibrium. For the system, averaged over a rapidly variable, we obtain parameters at which the stationary state is stable. Stabilization of the stationary state due to modulation of the delay can be successful (unsuccessful) in domains adjacent to super (sub-) critical Hopf bifurcation boundaries. In a vicinity of the bifurcation points, stable and unstable periodic solutions are asymptotically described in dependence on the modulation frequency.

1. Introduction

In recent years, the efficiency of dynamic control by using a variable delayed feedback (FB) is actively discussed. Schemes with modulation of the delay time [1–5] or of the FB level [6,7] have been considered. The idea of such control can be associated with the classical results on the dynamics of a pendulum with a vibrating suspension, so it is natural to expect stabilization of an unstable equilibrium due to high-frequency modulation of the FB parameters. In dynamical systems with delay, these effects and their application to vibration control were first considered on the base of the averaging theory in [8,9]. Numerical analysis has confirmed theoretical conclusions for some abstract models such as a linear oscillator [1], Lorentz [2] and Mackey–Glass [3] equations. In this paper we study the realistic optical model with parameters typical for class B lasers. We show that variable FB control can fail for some reasons common for delayed nonlinear systems.

In laser devices, optical or optoelectronic delayed feedback induces various instabilities in the form of periodic, quasi-periodic and chaotic pulsing. A lot of efforts have been paid to study such complex dynamics [10,11] as well as to developing methods for stabilization of stationary generation, for example, see [12] and references therein. Stabilization of unstable state by variable FB control has not been yet considered in lasers. Here we fill this gap for a rather simple model of a laser with incoherent optical FB.

Of particular importance is the coexistence of attractors, which may prevent the successful control in delayed systems. Multistability of periodic orbits is feature of a number of laser models with optoelectronic FB in the circuit pump [13,14] or loss [15], as well as models with purely optical coherent FB [16]. In general, domains of multistability cannot be described completely, even numerically, because the phase space of differential–difference systems is infinite dimensional. One possible way towards solving the problem

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http://dx.doi.org/10.1016/j.physd.2014.10.002
0167-2789/

was elaborated in [15,17,18] when studying the coexisting large-amplitude pulsed regimes of different periods.

Here we demonstrate that the oscillatory modes coexist with the stable stationary generation in parameter domains adjoined to subcritical Hopf bifurcation borders. In dependence on the initial state one can observe different regimes and hysteresis phenomena when parameters vary. Hence, the control can be successful (unsuccessful) in dependence on super- or subcritical Hopf bifurcation occurs. Other examples of coexisting attractors (cycles and tori) in a vicinity of the bifurcation points were discussed in [16,19,20] for lasers with optical FB.

The second aim of this work is to describe analytically characteristics of the periodic regimes induced by variable FB. Their dependence on the modulation frequency may be useful for applications in optical vibrometry [21].

The paper is organized as follows. In Section 2 we present the model proposed in [22] for a semiconductor laser with an incoherent optical FB. The model describes the laser dynamics in agreement with experimental observations. To verify the stabilizing effect of variable FB we introduce a periodic modulation of the time delay into the model. Periodic modulation of the time delay can be arranged by optoelectronic means or by a consequence of vibration of surface forming the external cavity.

The modulation frequency is assumed sufficiently large that allows us to average the original delayed system as presented in Section 3. In Section 4 a linear stability analysis is performed for the averaged system. It will be shown that the stability domains expand in the FB-parameter space in comparison with ones for the system with a constant delay. Furthermore there are bands of the modulation amplitude values at which the minimal FB level corresponding to Hopf bifurcation exceeds physically allowable level. In Section 5 we reduce the averaged system to a special ordinary differential equation. The obtained normal form determines, in main, system dynamics in the neighborhood of the stationary state in the bifurcation point. The direction of the bifurcation (super- or subcritical) will be determined. A possibility of stabilization of the stationary state in these conditions is discussed. In Section 6, the asymptotic (at high modulation frequency) approximation of limit cycles is presented in a vicinity of Hopf bifurcation point for the original non-autonomic system with variable delay. The dependence of cycle characteristics on modulation frequency is found.

2. Model

In the scheme of optical FB proposed in paper [22], the polarization of light reflected from the external mirrors is orthogonal to the polarization of incident light, that allows do not take into account interference effects in the mathematical model. Dynamics of generation is described by single-mode rate equations with delayed argument,

$$\begin{aligned} \dot{u} &= vu(y-1) \\ \dot{y} &= q - y - y[u + \gamma u(t-\tau)], \end{aligned} \tag{1}$$

where $u(t)$ and $y(t)$ are proportional to the density of photons and the population inversion, respectively, q is the pumping rate, v is the ratio of the rate of decay photons in the cavity to the rate of relaxation of populations, t and τ are current time and the radiation round-trip time in the external cavity normalized at the time of relaxation of the population inversion, γ is the feedback factor (level). For class B lasers including semiconductor lasers, some solid-state lasers, CO_2 gas lasers values $v \sim 10^3$ and $q >\sim 1$ are typical. The FB level is in the interval $\gamma \in [0, 1)$. The time delay τ can be varied from 0 to $\sim$10 and larger.

The system has two stationary states. The first state with zero radiation density, $u(t) = 0$, $y(t) = q$, becomes unstable when the pumping rate exceeds the threshold value $q = 1$. Next we assume that $q > 1$. When this occurs, the second equilibrium appears, $u(t) = u_s$, $y(t) = y_s$, where

$$u_s(\gamma) = \frac{q-1}{1+\gamma}, \qquad y_s = 1. \tag{2}$$

This stationary state is stable for $\gamma = 0$ and becomes unstable with increasing γ. Then periodic, quasiperiodic and chaotic oscillations are observed in the system [22].

Further we consider more complex system with periodically modulated time delay,

$$\begin{aligned} \dot{u} &= vu(y-1), \\ \dot{y} &= q - y - y[u + \gamma u(t - \tau - a\sin\omega t)], \end{aligned} \tag{3}$$

where ω is the frequency modulation and a characterizes the amplitude modulation which is limited by the inequality

$$|a| \le \tau.$$

The basic assumption, which opens the way to the application of asymptotic methods, is that the modulation frequency ω is sufficiently large compared to the intrinsic frequency,

$$\omega \gg 1. \tag{4}$$

Note, this assumption is rather difficult to implement in class B lasers as their characteristic frequencies are of the order of $v^{1/2}$. Such a large frequency can be provided by optoelectronics means.

System (3) has the same stationary states as system (1) with a constant delay. In the next part, we apply the averaging method to analyze the solutions near the equilibrium of system (3) with variable delay.

3. Averaged system

Let us recall some results for systems of ordinary differential equations with rapidly oscillating coefficients,

$$\dot{x} = f(\omega t, x), \tag{5}$$

where $\omega \gg 1$, and the vector-function $f(\Gamma, x)$ is periodic (almost periodic) by the first argument $\Gamma = \omega t$ and sufficiently smooth in the second argument. The behavior of solutions to Eq. (5) can be defined, in the main, by solutions of the averaged autonomous system

$$\dot{x} = f_0(x), \quad \text{where } f_0(x) = \lim_{T\to\infty} \frac{1}{T}\int_0^T f(\Gamma, x)d\Gamma. \tag{6}$$

The proof is based on the fact that system (5) takes the form

$$\frac{dx}{d\Gamma} = \omega^{-1} f(\Gamma, x), \tag{7}$$

in the result of the change of time variable $t = \omega^{-1}\Gamma$. Then, the derivative $\frac{dx}{d\Gamma}$ is quite small. For systems with delay and rapidly oscillating coefficients such time replacement is useless because the delay time increases unlimitedly with $\omega \to \infty$. However, under certain restrictions the principle of averaging can be also applied to delayed systems [8,9].

Consider system (3) following the above averaging procedure. Fix at an arbitrary point t_0 initial conditions $u(t_0 + \theta)$, $\theta \in [-\tau, 0]$ and $y(t_0)$ in the phase space $C_{[-\tau-a,0]} \times R^1$. Substitute them into the right part of system (3) and apply averaging over the fast time $\Gamma = \omega t$. Then we get the equations,

$$\begin{aligned} \dot{u} &= vu(y-1), \\ \dot{y} &= q - y - y\left[u + \gamma\frac{\omega}{2\pi}\int_0^{2\pi/\omega} u(t - \tau - a\cos(\omega\eta))d\eta\right]. \end{aligned}$$