



Modified inelastic bouncing ball model for describing the dynamics of granular materials in a vibrated container



Junius André F. Balista^{a,b}, Caesar Saloma^{a,*}

^a National Institute of Physics, University of the Philippines, Diliman 1011, Quezon City, Philippines

^b Institute of Mathematical Sciences and Physics, University of the Philippines, Los Baños, College 4031, Laguna, Philippines

ARTICLE INFO

Article history:

Received 9 May 2014

Accepted 3 October 2014

Available online 13 October 2014

Communicated by D. Lohse

Keywords:

Non-linear dynamics

Bifurcations

Bouncing

Granular flow

Convection

ABSTRACT

We show that at the onset of convection, the acceleration of a confined granular material is not necessarily equal to that of its vibrated container. Convection happens when the material is able to counter the downward gravitational pull and accelerates at a rate that is equal to the gravitational acceleration g . We modify the Inelastic Bouncing Ball Model and incorporate the transmissibility parameter T_r which measures the efficiency that the external force driving the container is transmitted to the material itself. For a specified T_r value, the material is represented by an inelastic bouncing ball with a time-of-flight $T(\Gamma; T_r)$ where $\Gamma = A_0\omega^2/g$, is the dimensionless container acceleration, and A_0 and ω are the driving amplitude and angular frequency, respectively. For a given Γ -range, the $T(\Gamma; T_r)$ curve provides the bifurcation diagram of the perturbed material and a family of bifurcation diagrams is generated for a set of T_r values. We illustrate that T_r is useful in rationalizing experimental results produced by confined granular materials that is subjected to a range of applied force magnitudes. For the same physical set-up, the force transmission efficiency from the container to the grains may not remain constant as the force strength is varied. The efficiency is also affected by the presence or absence of air in the vibrated container.

© 2014 Elsevier B.V. All rights reserved.

1. Introduction

An object that is in contact but not attached to a vertically oscillating container, levitates when its dimensionless acceleration $\Gamma = a/g$, becomes equal to unity ($\Gamma = 1$) where $a = A_0\omega^2$, is the container acceleration, g is the gravitational acceleration, and A_0 and ω are the vibration amplitude and angular frequency, respectively. The aforementioned threshold condition however, does not automatically apply to movements of extended objects such as confined granular materials that are undergoing convection [1–4]. Assigning a common acceleration for the confined grains and its container is inaccurate because the external force that is applied to the container is not always identical to the force that is experienced by the grains themselves due to possible transmission delays and modulation effects that are caused by elasticity and other dissipation properties.

Poeschel and Schwager [5] attributed the deviation of Γ from unity to the nonlinearity of the granular interactions. They studied

the motion of the individual grains using the response parameter R that depends on the elastic and dissipative properties of the grains. Okudaira [6] on the other hand, treated the granular material in bulk and utilized the transmissibility parameter T_r , that has a simpler functional form than R .

We re-examine the force-transmission issue and address the continuing interest at finding more accurate models for explaining the complex dynamics that is observed in granular materials, especially mixtures [7,8]. We investigate the potential of the Inelastic Bouncing Ball Model (IBBM) that was originally formulated to describe the motion of an inelastic ball on an oscillating plate [9–11]. The IBBM stipulates that the ball levitates once its upward acceleration exceeds g and dissipates its kinetic energy completely upon collision with the plate. Pastor et al. [12] had suggested earlier that the IBBM at least in its original form, is inadequate for describing the dynamics of a vibrated granular material.

We show that at the onset of convection, the acceleration of the confined granular material is not necessarily equal to that of its vibrated container. We extend the applicability of the IBBM to perturbed confined granular systems by modifying the IBBM and incorporating the transmissibility parameter T_r which measures the efficiency that the external force driving the container is transmitted to the material itself.

* Corresponding author. Tel.: +63 29295401.

E-mail addresses: caesar.saloma@gmail.com, csaloma@nip.upd.edu.ph (C. Saloma).

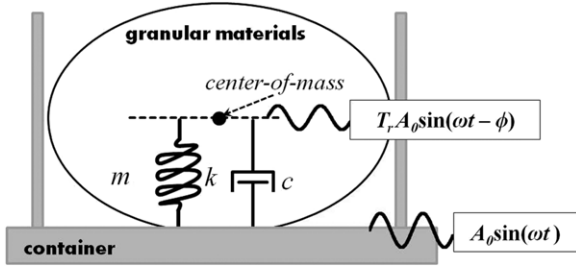


Fig. 1. Schematic diagram of the granular material and the container before granular levitation. The granular material is considered as a single object with mass m , effective spring constant k and damping coefficient c . The vertical oscillation, of amplitude A_0 and angular frequency ω , is transmitted from the container to center-of mass of the granular material at transmissibility T_r .

The material is represented by an inelastic bouncing ball that produces a time-of-flight $T(\Gamma; T_r)$. For a given T_r and Γ -range, $T(\Gamma; T_r)$ provides the bifurcation diagram of the perturbed material and a family of bifurcation diagrams is produced for a set of T_r values. At the onset of convection, Γ is equal to unity (i.e., $a = g$) only when the force transmission is instantaneous and lossless ($T_r = 1$). We illustrate that T_r is a useful tuning parameter for rationalizing experimental results where the T_r value is not held constant within the Γ range for the same physical set-up.

Our presentation proceeds as follows: In Section 2, we derive the expression for the time-of-flight $T(\Gamma; T_r)$ and establish the critical acceleration a_c of the container in terms of T_r at the instant of granular separation and in Section 3, we describe the modified IBBM and compare its predictions with published experimental results using T_r as a tuning parameter.

2. Critical acceleration of container

The practical way towards understanding the kinetics of a perturbed confined granular system is to track the motion of its center-of-mass (CM) and not the individual movements of the grains [12]. We now determine the critical acceleration a_c of the container when the CM separates from the container. The subscript c is introduced to emphasize that a_c refers to the critical acceleration of the container and not that of the CM of the granular materials.

For simplicity, the granular material is considered as a continuous object with mass m . The grains connecting its CM to the container are treated as a spring system with effective spring constant k and damping coefficient c . Fig. 1 presents the schematic diagram of the system before levitation of the granular materials where only motions parallel to gravity are considered. The interactions between the grains and the container walls are assumed to be either negligible for containers with low aspect ratio [12] or subsumed by the transmissibility parameter T_r , which is explained below.

We treat the vibration of the granular material as a vibration-isolation problem with a single degree of freedom, which also allows us to utilize the concept of T_r [13]. In the linear approximation regime, the transmissibility may be defined as: $T_r T_r = F_T / F_I$, where F_T is the force (magnitude) transmitted to the CM while F_I is the (external) force applied to the container. We point out that concept of T_r was employed in confined granular systems before [6,14–16] but without specifying the form of the critical acceleration a_c .

We consider a container that is vertically oscillated according to: $A_0 \sin(\omega t)$, with the laboratory as reference frame. The corresponding vertical displacement of the granular material before its separation is: $T_r A_0 \sin(\omega t - \phi)$, where $\phi = \arctan(2\zeta\omega/(\omega_0^2 - \omega^2))$, $\omega_0^2 = k/m$ and $\zeta = c/(2\sqrt{km})$ correspond to the phase delay, resonance angular frequency, and damping ratio, respectively. The

phase delay ϕ is introduced since force transmission from the container to the CM does not happen instantaneously. The transmissibility is given by [13]:

$$T_r = \sqrt{\frac{\omega_0^4 + (2\zeta\omega\omega_0)^2}{(\omega_0^2 - \omega^2)^2 + (2\zeta\omega\omega_0)^2}}. \quad (1)$$

If the container oscillation cycle starts at $t = 0$, then the granular material separates from the container at later time $t = t_0$ when the following condition is satisfied:

$$T_r A_0 \omega^2 \sin(\omega t_0 - \phi) = g. \quad (2)$$

The threshold condition specified in Eq. (2) is different from the one stated previously in Ref. [4]: $A_0 \omega^2 \sin(\omega t) = g$, which considers the granular material and its container as a single entity. An expression for the separation time t_0 may be derived from Eq. (2) as:

$$t_0 = \frac{1}{\omega} \left[\arcsin\left(\frac{g}{T_r A_0 \omega^2}\right) + \arctan\left(\frac{2\zeta\omega}{\omega_0^2 - \omega^2}\right) \right]. \quad (3)$$

With Eq. (3), the acceleration a_c and the dimensionless acceleration Γ may be expressed at $t = t_0$, as:

$$\begin{aligned} a_c &= A_0 \omega^2 \sin(\omega t_0) \\ &= \left[\frac{g}{T_r} + \sqrt{\left(\frac{2A_0 \omega^3 \zeta}{\omega_0^2 - \omega^2}\right)^2 - \left(\frac{2\omega\zeta}{\omega_0^2 - \omega^2}\right)^2 \left(\frac{g}{T_r}\right)^2} \right] \\ &\quad \times \sqrt{\frac{(\omega_0^2 - \omega^2)^2}{(\omega_0^2 - \omega^2)^2 + (2\zeta\omega)^2}} \neq g \end{aligned} \quad (4)$$

$$\Gamma = \frac{a_c}{g} \neq 1. \quad (5)$$

We note that: $a_c = g$ or $\Gamma = 1$, only when the force transfer is lossless and instantaneous ($T_r = 1$) at the time ($t = t_0$) when the grains separate from the container wall.

The dimensionless acceleration Γ of the container is a useful variable to employ in experiments since its values are easy to measure accurately—an accelerometer is simpler to be attached to the container wall than to the CM of the confined granular material.

3. Bifurcation diagram of the time-of-flight

We apply the modified IBBM to the case of vertically vibrated granular layers and derive an expression for its time-of-flight $T(\Gamma; T_r)$ in terms of T_r and Γ . The $T(\Gamma; T_r)$ curve provides the bifurcation diagram of the granular material.

Earlier, Melo et al. [17] reported the formation of interesting patterns in vertically vibrated granular layers and found correlations between the pattern characteristics and the features displayed by the corresponding experimental time-of-flight $T(\Gamma)$ plot within the range: $1 \leq \Gamma \leq 8$. The $T(\Gamma)$ plot was recognized to correspond to the bifurcation diagram of an inelastic bouncing ball but with the following distinguishing features: (1) Times-of-flight were generally shorter and the experimental Γ values at the bifurcation points were larger than predicted; and (2) Saddle-node bifurcation (plateau) features were not apparent in the experimental results. The nonzero $T(\Gamma)$ values in both the experimental and theoretical diagrams started at $\Gamma = 1$.

About a decade later, Pastor et al. [12] performed similar experiments on granular layers to produce bifurcation diagrams over a shorter range of: $1 \leq \Gamma \leq 6$. In addition to validating the two previous observations of Melo et al., they also noticed bifurcation branches that were not divergent from each other.

Download English Version:

<https://daneshyari.com/en/article/1898360>

Download Persian Version:

<https://daneshyari.com/article/1898360>

[Daneshyari.com](https://daneshyari.com)