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Qualitative analysis of the Rössler equations: Bifurcations of limit cycles and chaotic attractors

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ABSTRACT

In this paper we study different aspects of the paradigmatic Rössler model. We perform a detailed study of the local and global bifurcations of codimension one and two of limit cycles. This provides us a global idea of the three-parametric evolution of the system. We also study the regions of parameters where we may expect a chaotic behavior by the use of different Chaos Indicators. The combination of the different techniques gives an idea of the different routes to chaos and the different kinds of chaotic attractors we may found in this system.

1. Introduction

Within the field of dynamical systems and chaos for low-dimensional systems, two paradigmatic problems that have been frequently studied are the Lorenz and the Rössler [1] models. Both of them have attracted a large number of studies and they continue to appear in the literature [2–9]. The main reasons are that they are well known but not completely understood, and as paradigmatic problems they have become test problems for almost all new analytical and numerical techniques in computational dynamics.

The Rössler equations [1] are given by

$$\begin{aligned}\dot{x} &= -(y+z),\\ \dot{y} &= x+ay,\\ \dot{z} &= b+z(x-c),\end{aligned} \tag{1}$$

with $a, b, c \in \mathbb{R}$, and they are assumed to be positive and dimensionless. This model is a famous prototype of a continuous dynamical system exhibiting chaotic behavior with minimum ingredients.

As is well known, the Rössler equations have two equilibrium points P_1 and P_2 for $c^2 > 4ab$ given by $P_1 = (-ap_1, p_1, -p_1)$ and $P_2 = (-ap_2, p_2, -p_2)$ with

$$p_1 := \frac{1}{2}\left(-\frac{c}{a} - \frac{\sqrt{c^2-4ab}}{a}\right), \quad p_2 := \frac{1}{2}\left(-\frac{c}{a} + \frac{\sqrt{c^2-4ab}}{a}\right).$$

Clearly the conditions of the existence of the equilibria $b_E \equiv b = c^2/(4a)$ give a surface of fold (saddle-node or tangent) bifurcations. These equilibria have several bifurcations [5,10,11], as different curves of Andronov–Hopf bifurcation around P_1 and P_2. This bifurcation gives one mechanism of creation of limit cycles around equilibria. In this paper we focus our attention on a detailed analysis of the bifurcations of the limit cycles, which will give us a global idea of the parameter evolution of the system and the different kinds of chaotic attractors we may find in the parametric phase space.

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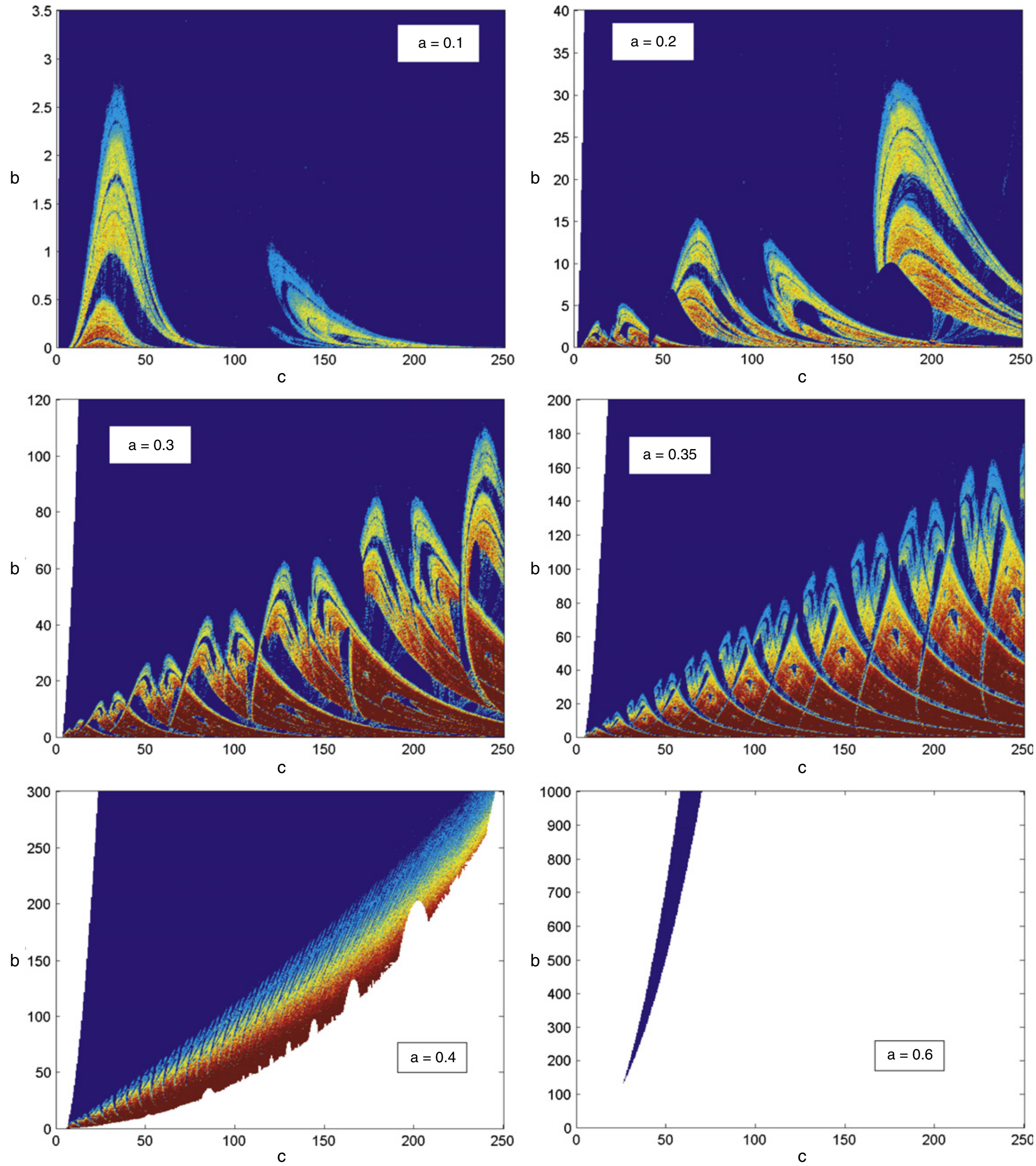

Fig. 1. BPD diagrams on the (c, b) plane for different values of the parameter a. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

This paper is organized as follows. In Section 2 we study the different regions in the space of parameters depending on the behavior of the orbits, regular or chaotic. This is done with the combined use of different Chaos Indicators [12–15]. Later, in Section 3 we study numerically, by the use of the well-known softwares `AUTO` [16,17] and `MATCONT` [18], the local bifurcations of limit cycles far from the local bifurcations of equilibria. Moreover, we show these bifurcations over some of the Chaos Indicator plots and we present some Poincaré sections to study the different coexisting structures. This permits us to propose a generic scheme of the parametric phase space. In Section 4 we study the different routes to chaos and the different kinds of chaotic attractors we may find and where they are located in the parametric phase space, and finally, in Section 5 we present some global bifurcations, and how they organize the complete parametric phase space.

2. Regular and chaotic regions

The Rössler model is a three-dimensional problem with three dimensionless parameters. It is well known that for different sets of values of the parameters the model exhibits a chaotic behavior, while for others a regular behavior. Therefore, a study of the different regions in the parameter space will give some interesting results. To do this we use a combination of different numerical techniques. To be precise, we use as main techniques the Maximum Lyapunov Exponent (MLE) [19], computed by using the algorithm given in [15], and the OFLI2 Chaos Indicator, proposed in [13]. We have used these methods to perform a systematic search in different planes of parameters in order to locate the values of the parameters that give a probable chaotic behavior. A positive value of the MLE is usually associated with a chaotic be-