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The random attractor of stochastic FitzHugh–Nagumo equations in an infinite lattice with white noises☆

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Abstract

The present paper is devoted to the existence of the random attractor of stochastic FitzHugh–Nagumo equations in an infinite lattice with additive white noise. Using the Ornstein–Uhlenbeck transform, we firstly show the existence of an absorbing set, then prove that the random dynamical system is asymptotically compact. Finally, the existence of the random attractor is provided.

Keywords: Random attractor; Additive white noise; Stochastic FitzHugh–Nagumo systems

1. Introduction

The current paper is devoted to the study of asymptotic behaviors for stochastic FitzHugh–Nagumo equations in infinite lattices with additive white noise. FitzHugh–Nagumo equations are used to describe the signal transmission across axons. There are many papers investigating the global attractor and its Hausdorff dimensions (see [12] and the references therein), and the existence and stability of traveling wave solutions (see [7,8,6,9] and the references therein). Recently, lattice dynamical systems have been extensively studied; for the traveling wave solution, we refer the reader to [11]. For the attractor and its upper semi-continuity, we refer the reader to [14,13]. The random attractor of stochastic lattice dynamical systems has been studied; we refer the reader to [2,10].

The authors in [9] studied the following FitzHugh–Nagumo neurons with white noise:

$$\begin{cases} \dfrac{\mathrm{d}x_i}{\mathrm{d}t} = c\left(x_i - \dfrac{1}{3}x_i^3 + y_i\right) + h\displaystyle\sum_{k\in Z}\delta(t - 2k\pi/\omega) - \dfrac{d}{N-1}\sum_{j=1}^{N}(x_i - x_j) + D_i\eta_i(t), \\ \dfrac{\mathrm{d}y_i}{\mathrm{d}t} = -\dfrac{1}{c}(x_i + by_i + a), \quad i = 1, 2, \ldots, N \end{cases} \tag{1.1}$$

where $\eta_i(t)$ is the Gaussian white noise, and investigated the influence of noise on synchronization between the spiking activities of neurons with external impulsive forces by means of numerical simulations.

Motivated by [2,13], we study the asymptotic behaviors of stochastic FitzHugh–Nagumo Equations with white noises on an infinite lattice as follows:

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$$\begin{cases} \dfrac{\mathrm{d}u_i}{\mathrm{d}t} = u_{i+1} - 2u_i + u_{i-1} + h(u_i) - v_i + a_i \dfrac{\mathrm{d}w_i^1(t)}{\mathrm{d}t}, \\ \dfrac{\mathrm{d}v_i}{\mathrm{d}t} = \sigma u_i - \delta v_i + b_i \dfrac{\mathrm{d}w_i^2(t)}{\mathrm{d}t} \end{cases} \tag{1.2}$$

where $i \in Z, u = (u_i)_{i \in Z} \in l^2$ and $v = (v_i)_{i \in Z} \in l^2$, σ and δ are positive constants, $\{w_i | i \in Z\}$ are independent Brownian motions, and show the existence of a random attractor for systems (1.2).

The rest of the paper is organized as follows. In Section 2, we introduce definitions and the standing hypothesis. The existence of random attractors for (1.2) on infinite lattices is shown in Section 3.

2. Preliminary

In this section, we introduce the definitions of the random dynamical system and random attractor, which are taken from [3–5,2]. Let (H, d) be a complete separable metric space, $(\Omega, \mathbb{F}, P)$ be a probability space.

Definition 2.1. $(\Omega, \mathbb{F}, P, (\theta)_{t \in R})$ is called a metric dynamical system if $\theta : R \times \Omega \to \Omega$ is $(B(R) \times \mathbb{F}, \mathbb{F})$ measurable, $\theta_0 = I, \theta_{s+t} = \theta_s \circ \theta_t$ for all $t, s \in R$, and $\theta P = P$ for all $t \in R$.

Definition 2.2. A random dynamical system (RDS) with time T on a metric, complete and separable space (H, d) with Borel σ-algebra $\mathcal{B}$ over $\{\theta_t\}$ on $(\Omega, \mathbb{F}, P)$ is a measurable map

$$S : T \times H \times \Omega \longmapsto H, \qquad (t, s, \omega) \longmapsto S(t, \theta_{-t}\omega)x$$

such that

(i) $S(0, \omega) = \mathrm{Id}$ (identity on H), and
(ii) (*Cocycle property*) $S(t + s, \omega) = S(t, \theta_s \omega) \circ S(s, \omega)$ for all $s, t \in T$ and $\omega \in \Omega$.

Definition 2.3. An RDS is said to be continuous or differentiable if $S(t, \theta_{-t}\omega) : H \longmapsto H$ is continuous or differentiable respectively, for all $t \in T$ outside a P-nullset. Set $B \subset \Omega$ is called invariant with respect to $(\theta_t)_{t \in R}$ if for all $t \in R$, $\theta_t^{-1} B = B$.

Definition 2.4. A random set $K(\omega)$ is said to be $S(t, \theta_{-t}\omega)$ forward invariant if

$$S(t, \theta_{-t}\omega)K(\theta_{-t}\omega) = K(\omega).$$

Definition 2.5. For a given random set $B(\omega)$, the set

$$\Omega(K(\omega), \omega) = \Omega_K(\omega) = \bigcap_{s \geq 0} \overline{\bigcup_{t \geq s} S(t, \theta_{-t}\omega)K(\theta_{-t}\omega)}.$$

There is an equivalent definition,

$$\Omega_K(\omega) = \{y \in H : \text{ there exist } t_n \to \infty \text{ and } x_n \in K(\theta_{-t_n}\omega) \text{ such that } S(t_n, \theta_{-t_n})x_n \to y, \text{ as } n \to \infty\}.$$

The θ-shift of an Ω-limit set is

$$\Omega_K(\cdot) \cdot \theta_t = \Omega(K, \theta_t \omega) = \{y \in H : \text{ there exist } t_n \to \infty \text{ and } x_n \in K(\theta_{-t_n+t}\omega) \text{ such that } S(t_n, \theta_{-t_n+t})x_n \to y, \text{ as } n \to \infty\}.$$

Definition 2.6. A random set $A(\omega)$ is said to attract another random set $B(\omega)$ if P-almost surely,

$$d(S(t, \theta_{-t}\omega)B(\theta_{-t}\omega), A(\omega)) \to 0, \quad \text{as } n \to \infty.$$

Definition 2.7. If $K(\omega)$ and $B(\omega)$ are random sets such that for P-almost all $\omega \in \Omega$, there exists a time $t_B(\omega)$ such that for all $t \geq t_B(\omega)$

$$S(t, \theta_{-t}\omega)B(\theta_{-t}\omega) \subset K(\omega),$$

then $K(\omega)$ is called as an absorbing set with respect to $B(\omega)$, and $t_B(\omega)$ is called the absorption time.

Definition 2.8. Suppose $S(t, \omega)$ is an RDS such that there exists a random compact set $\omega \to \mathcal{A}(\omega)$ which satisfies the following conditions:

(i) $S(t, \omega)\mathcal{A}(\omega) = \mathcal{A}(\theta_t \omega)$ for all $t > 0$, and
(ii) $\mathcal{A}(\omega)$ attracts every bounded deterministic set $B \subset H$.

Then $\mathcal{A}(\omega)$ is called a global random attractor.