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A novel mathematical analysis for predicting master–slave synchronization for the simplest quadratic chaotic flow and Ueda chaotic system with application to communications

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Abstract

A master–slave concept of chaotic system synchronization is examined and applied in secure data transmission. In most situations when examining this type of synchronization one considers the asymptotic stability of the particular system via Lyapunov's direct method, or conditional Lyapunov exponents are considered. In this paper Lyapunov's direct method is employed to show the asymptotic stability within the simplest piecewise linear master–slave chaotic flow. Also, however, in this paper, primarily the master–slave synchronization properties of the simplest quadratic chaotic flow and Ueda chaotic system are examined directly by means of mathematical manipulation of their dynamics equations, where possible, as well as via numerical simulations. In order to achieve this, in conjunction, numerical simulations and theoretical analysis are made use of. In this way it is shown that the synchronization error of the two aforementioned chaotic master–slave systems can indeed be predicted for certain driving signals, without the need for either analytical or numerical evaluation of the conditional Lyapunov exponents or employment of Lyapunov's direct method. Finally, a digital communication system based on the initial condition modulation of the chaotic carrier by the binary message to be transmitted, in the presence of noise, is proposed and evaluated for the three chaotic systems investigated.

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1. Introduction

The problem of chaotic synchronization was first studied by Yamada and Fujisaka in 1983 [1], and by Afraimovich et al. in 1986 [2]. When Pecora and Carroll introduced their chaos synchronization method in 1990 [3], now commonly termed the Pecora–Carroll (PC) method, the topic of chaotic synchronization started to arouse major interest. In the PC method one has a master system and a slave system, with a single signal of the master system driving the slave system [3–10]. Similar master–slave synchronization schemes have also been considered elsewhere [11,12]. Besides the PC synchronization method, numerous chaos synchronization methods have been developed in the last decade and a half, such as the Ott–Grebogi–York (OGY) based chaos synchronization method [13,14], the John and Amritkar (JA) synchronization method [15], and Pyragas's synchronization method [16]. In more general terms the chaotic synchronization phenomena can be divided into identical synchronization (IS) and general synchronization (GS), among other types [17]. IS, as the name suggests, involves two identical systems, whereas generalized synchronization is an extension of IS, involving non-identical systems [17,18]. However it has been shown that in fact identical systems can also exhibit GS, thus proving that nonidentity of the systems is not a necessary condition for GS [17].

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The motivation for the study of chaotic synchronization lies in its numerous potential applications. The applications of chaotic synchronization range from living systems applications [19,18] to non-living systems applications [18,20]. Perhaps the most popular application of chaotic synchronization is in communications. Since the onset of chaotic synchronization research, a number of demodulation techniques based on chaotic synchronization have been proposed for potential communication systems—for instance, see [8,9,15, 21–28], of which the following are based on the PC synchronization method: [8,9,21–23,25,28].

In their original paper on chaotic synchronization [3], Pecora and Carroll suggested the application of chaos synchronization in communications, and shortly after Oppenheim et al. presented a communication system based on the PC synchronization method [21]. The method of [21], termed "chaotic masking", was experimentally demonstrated in [22] using Chua's circuit. In this method an information signal is added onto the chaotic carrier directly, and transmitted. The requirement of the method is for the power of the information signal to be significantly lower than the power of the chaotic carrier [21].

In contrast to chaotic masking, a communication technique based on PC synchronization but utilizing parameter modulation to encode the message has been introduced in [25]. In this technique, termed "chaotic modulation", the parameter b of the Lorenz chaotic flow has been varied between 4 and 4.4 depending on whether a binary 0 or a binary 1 is to be transmitted. On the receiver (slave) side the parameter b is fixed at 4 so that the synchronization either occurs or does not occur depending on the state of the parameter b on the transmitter (master) side.

Communication methods based on chaotic synchronization other than PC synchronization have also been proposed. For instance in [24] chaotic masking and Pyragas's synchronization method have been used to transmit and receive information, whereas in [15] chaotic modulation and the JA synchronization method have been used.

In [3] it has been demonstrated that the necessary condition for PC master–slave synchronization to occur is for the sub-Lyapunov exponents (later renamed conditional Lyapunov exponents [4]) of the non-driving/non-driven subsystem to be less than zero. In particular, this has been shown for the Lorenz and Rossler chaotic systems [3]. The necessary and sufficient condition for master–slave synchronization to occur is that the part of the slave system not being driven by the master system must be asymptotically stable [7]. Asymptotic stability of a system can be demonstrated via Lyapunov's direct (or second) method [29] by demonstrating the existence of the Lyapunov function. This method has been used to show that the Lorenz master–slave systems must synchronize when the master x signal drives the slave system [7]. Using the methodology from [7] a similar proof has been derived for the Van der Pol–Duffing oscillator in [8]. In this paper, initially the existence of the Lyapunov function is briefly demonstrated, following the procedure of [7,8], for the simplest piecewise linear chaotic flow, when the x signal is driving. The simplest piecewise linear chaotic flow is easily realizable in the form of an electronic circuit [30]. However, the main emphasis of this paper is on the master–slave synchronization via direct mathematical analysis of the dynamics of the simplest quadratic master–slave chaotic flow and the Ueda master–slave chaotic system. These systems have been selected for the analysis due to it being possible, as shown in this paper, to analyse their PC synchronization properties without the use of Lyapunov's stability theory or the need to obtain the conditional Lyapunov exponents. Numerical simulations are used to further support the analysis.

To demonstrate the potential application of the chaotic synchronization phenomena described in this paper, a simple communication system is presented where the message modulates the chaotic signal by changing the initial conditions of one of the master signals. In addition, the robustness against noise of this system is also examined.

Section 2 explains the concept of master–slave synchronization. In Section 3 it is shown that the asymptotic stability, which is a necessary and sufficient condition for synchronization, exists within the simplest piecewise linear master–slave chaotic flow when the master x signal drives the slave subsystem. This is followed by numerical simulations and a mathematical analysis of the simplest quadratic master–slave chaotic flow in Section 4 and the Ueda master–slave chaotic system in Section 5. In Section 6 a "chaotic synchronization" communication system, based on the initial condition modulation of the message to be transmitted, is proposed. This system utilizes the results obtained in Sections 3–5, and thus demonstrates the potential application of the synchronization results obtained in this paper.

2. Master–slave synchronization concept

Pecora and Carroll [3] were among the first to introduce the concept of synchronization in chaotic systems. Their synchronization scheme can be viewed as a master–slave system [7]. Essentially, a master–slave system consists of two chaotic systems. These systems are of the same type, with the same parameter values. For it to be possible for synchronization to occur, the output from at least one of the coupled differential equations of the first chaotic system must be made available to the second chaotic system. Thus, one chaotic system is said to drive the other chaotic system via the time-series signal generated from one of its differential equations. The driving chaotic system is known as the master system and the driven chaotic system is known as the slave system. Together, these systems form a master–slave system. Such a system is depicted in Fig. 1.

The master system is made up of a driving master subsystem (u) with an initial condition $u(0)$ and a non-driving master subsystem (v) with initial conditions $v(0)$ which are independent of the master driving subsystem. The slave system is made up of a driven slave subsystem ($\hat{u}$), which is identical to (u), and a non-driven slave subsystem ($\hat{v}$), which has initial conditions $\hat{v}(0)$. Since the driving master subsystem is fully made available to the slave system, it is said that the master system drives the slave system with the driving master