MAGNETIC CURVES IN COSYMPLECTIC MANIFOLDS

SIMONA-LUIZA DRUŢĂ-ROMANIUC

Department of Mathematics and Informatics,
'Gheorghe Asachi' Technical University of Iasi,
Bd. Carol I no. 11, 700506 Iasi, Romania
(e-mail: simona.romaniuc@tuiasi.ro)

JUN-ICHI INOGUCHI[a]

Department of Mathematical Sciences, Yamagata University,
Yamagata, 990-8560, Japan
Institute of Mathematics, University of Tsukuba,
1-1-1 Tennodai Tsukuba, Ibaraki, 305-8571, Japan
(e-mail: inoguchi@math.tsukuba.ac.jp)

MARIAN IOAN MUNTEANU[b]

'Alexandru Ioan Cuza' University of Iasi, Faculty of Mathematics,
Bd. Carol I, no. 11, 700506 Iasi, Romania
(e-mail: marian.ioan.munteanu@gmail.com)

and

ANA IRINA NISTOR[c]

Department of Mathematics and Informatics,
'Gheorghe Asachi' Technical University of Iasi,
Bd. Carol I no. 11, 700506 Iasi, Romania
(e-mail: ana.irina.nistor@gmail.com)

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In this paper we classify the magnetic trajectories with respect to contact magnetic fields in cosymplectic manifolds of arbitrary dimension. We classify Killing magnetic curves in product spaces $\mathbb{M}^2 \times \mathbb{R}$, recalling also explicit description of magnetic curves in $\mathbb{E}^3$, $\mathbb{S}^2 \times \mathbb{R}$ and $\mathbb{H}^2 \times \mathbb{R}$. Finally, we prove a reduction theorem for magnetic curves in the cosymplectic space form $\bar{M}^{2n}(k) \times \mathbb{R}$, in order to show that the (2n+1)-dimensional case reduces to the 3-dimensional one.

Keywords: contact magnetic field, magnetic curve, cosymplectic manifold.

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1. Introduction

It is worth mentioning the progress of research in different topics on almost contact geometry, either for theoretical, or for practical considerations. More precisely, remarkable is wide applicability of contact geometry in many problems of physics, such as the geometric description of time-independent mechanics [13]. Closely related to this topic, more precisely to magnetostatics, we deal with the study of time-independent magnetic fields and their corresponding magnetic curves on manifolds, that is, then depend only on the coordinates on the manifold.

On a (complete) Riemannian manifold (M, g) a closed 2-form F is called a *magnetic field*, since it can be regarded as a generalization of static magnetic fields in the three-dimensional Euclidean space [15, 24]. For a dynamical system (M, g, F) associated to the magnetic field F one can define the *Lorentz force* ϕ as the skew-symmetric $(1, 1)$-type tensor field corresponding to F via g. A *magnetic curve* corresponding to the magnetic field F is a smooth curve γ, usually parametrized by arc-length, which satisfies the *Lorentz equation* (also called the *Newton equation*) $\nabla_{\dot\gamma}\dot\gamma = q\phi\dot\gamma$, where ∇ denotes the Levi-Civita connection on M and q is a real constant called the *strength* of the magnetic field. Thus, we may say that the magnetic curve γ describes the trajectory of a charged particle moving under the action of the Lorentz force ϕ generated by the magnetic field F in the magnetic background (M, g, F). If $F = 0$, i.e. the Lorentz force is null, then the particles trajectories are geodesics. In this manner we may regard the magnetic curves as generalizations of geodesics.

In the 3-dimensional case one can easily understand the relation between magnetic fields and almost contact metric manifolds in the following manner. Denoting by (M, g) an oriented 3-dimensional Riemannian manifold endowed with the volume form dv_g, it is known that the space of all smooth 2-forms is identified with the space of all smooth vector fields via the Hodge star operator. Let now F denote a magnetic field on (M, g, dv_g), V its corresponding divergence free vector field and ω the dual 1-form of V with respect to the metric g. If V is unitary, then one can show that (ϕ, V, ω) is an almost contact structure on M, compatible with the metric g. In other words, a dynamical system (M, g, F), given by an oriented 3-dimensional Riemannian manifold (M, g) together with a magnetic field F having the corresponding divergence-free vector field unitary, may be thought as an almost contact metric manifold with closed fundamental 2-form [5]. For this reason we turn our attention to magnetic curves corresponding to magnetic fields in almost contact metric manifolds.

The almost contact metric manifolds were completely classified in [14]. An important class of these manifolds are the quasi-Sasakian manifolds, which were introduced by Blair in his Ph.D Thesis [8] (see also [9]), as normal almost contact metric manifolds having closed fundamental 2-form. The Sasakian and cosymplectic manifolds are quasi-Sasakian manifolds of rank $2n + 1$ and 1, respectively. Recall that the rank of the quasi-Sasakian structure is the rank of the 1-form η, i.e. η has *rank* equal to $2p$ if $(d\eta)^p \neq 0$ and $\eta \wedge (d\eta)^p = 0$, or η has *rank* equal to $2p + 1$