Physica D 331 (2016) 27–47

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Breathers in a locally resonant granular chain with precompression

Lifeng Liu [a], Guillaume James [b], Panayotis Kevrekidis [c,d], Anna Vainchtein [a,*]

[a] Department of Mathematics, University of Pittsburgh, Pittsburgh, PA 15260, USA
[b] INRIA Grenoble - Rhône-Alpes, Bipop Team-Project, Inovallée, 655 Avenue de l'Europe, 38334 Saint Ismier Cedex, France
[c] Department of Mathematics and Statistics, University of Massachusetts, Amherst, MA 01003, USA
[d] Theoretical Division and Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

HIGHLIGHTS

- A precompressed locally resonant granular chain is considered.
- An effective nonlinear Schrödinger modulation equation is derived.
- Alternating parametric intervals of dark and bright breathers are identified.
- Stability of stationary dark breathers is investigated.

ARTICLE INFO

Article history:
Received 19 January 2016
Received in revised form
16 May 2016
Accepted 17 May 2016
Available online 24 May 2016
Communicated by B. Sandstede

Keywords:
Locally resonant granular material
Precompression
Discrete breather
Modulation equation
Stability

ABSTRACT

We study a locally resonant granular material in the form of a precompressed Hertzian chain with linear internal resonators. Using an asymptotic reduction, we derive an effective nonlinear Schrödinger (NLS) modulation equation. This, in turn, leads us to provide analytical evidence, subsequently corroborated numerically, for the existence of two distinct types of discrete breathers related to acoustic or optical modes: (a) traveling bright breathers with a strain profile exponentially vanishing at infinity and (b) stationary and traveling dark breathers, exponentially localized, time-periodic states mounted on top of a non-vanishing background. The stability and bifurcation structure of numerically computed exact stationary dark breathers is also examined. Stationary bright breathers cannot be identified using the NLS equation, which is defocusing at the upper edges of the phonon bands and becomes linear at the lower edge of the optical band.

1. Introduction

Granular crystals are tightly packed arrays of solid particles that deform elastically upon contact via nonlinear Hertzian interactions [1–3]. The dynamics of these systems ranges from *weakly nonlinear*, when the initial overlap of the neighboring particles due to the static precompression is much larger than their relative displacement, to the *strongly nonlinear* regime characterized by relatively small or zero precompression. This provides an ideal setting for exploring nonlinear waves, including traveling [1–3] and shock waves [4,5].

A particularly interesting class of nonlinear excitations exhibited by these materials are the so-called *discrete breathers* [6–14], i.e., time-periodic and exponentially localized in space oscillations that are also encountered in a wide variety of other nonlinear systems (see [15,16] and references therein). There are two distinct types of breathers. *Bright breathers* have a profile (of strain in the case of granular systems) exponentially decaying to zero at infinity and are known to exist in granular materials with defects [11,13], heterogeneous granular chains such as dimers or trimers [6,12,14] and Hertzian chains with a harmonic onsite potential modeling Newton's cradle or granular chains embedded in a matrix [9,10,17]. *Dark breathers*, on the other hand, are spatially modulated standing waves with amplitude that is constant at infinity and vanishes at the center. They have been recently identified and analyzed in a homogeneous granular chain with precompression [7], and their existence was experimentally verified in damped, driven granular chains in [8].

In this work we consider both types of discrete breathers in a locally resonant granular chain characterized by very rich nonlinear dynamics [18,19]. This novel granular metamaterial has tunable band gaps and can be potentially used in engineering

* Corresponding author. Tel.: +1 412 624 8309; fax: +1 412 624 8397.
E-mail address: aav4@pitt.edu (A. Vainchtein).

http://dx.doi.org/10.1016/j.physd.2016.05.007
0167-2789/

applications involving shock absorption and vibration mitigation. The system consists of a regular granular chain with additional degrees of freedom due to attached linear resonators. Its recent experimental implementations include mass-in-mass granular chains with internal linear resonators placed inside the primary beads [20], mass-with-mass chains with external ring resonators attached to the beads [21] (see also [22]) and woodpile phononic crystals consisting of vertically stacked slender cylindrical rods in orthogonal contact [23]. Under certain assumptions, each of these experimental setups can be modeled by a Hertzian chain with a secondary mass attached to each primary bead by a linearly elastic spring, with the ratio of the secondary and primary masses being the main control parameter. In a recent work [24], we studied the strongly nonlinear dynamics of this system in the absence of precompression. Through a combination of asymptotic analysis and numerical computations, we provided evidence for the existence of exact dark breathers in the locally resonant granular chain and investigated their stability and bifurcation structure. In addition, we studied small-amplitude periodic traveling waves and identified the conditions under which the system has long-lived (but not exact) bright breathers.

Here we turn our attention to the locally resonant granular chain under nonzero precompression. In the non-resonant limit (regular granular chain, zero mass ratio), such a system belongs to the general class of Fermi–Pasta–Ulam (FPU) lattice models (e.g. see [25–35] and references therein), with a dispersion relation for plane wave solutions of the linearized problem possessing only acoustic spectrum. At finite mass ratio, the dispersion relation has both acoustic and optical branches. In this respect the problem is somewhat reminiscent of diatomic FPU chains, although the optical branch is quite different in our case. In the small-amplitude limit the dynamics of the system is weakly nonlinear. This dynamical regime has been well studied for the FPU problem, as has its generalized version with an additional onsite potential [16,27–29,31–34,36,37]. In particular, the established conditions for bifurcation of discrete breathers for this class of problems [16,32,33] rule out the existence of bright breathers in the homogeneous non-resonant granular chain under precompression, the limiting case of our problem when the mass ratio is zero and the dispersion relation has only an acoustic branch. In this case, dark breathers were identified in [7] as the *only* possible type of intrinsically localized mode. The *defocusing* nonlinear Schrödinger equation (NLS), which has tanh-type solutions, is derived in [7] as the modulation equation for waves with frequencies near the edge of the linear acoustic spectrum and used to construct initial conditions for numerical computation and analysis of the dark breathers. In another limiting case, when the mass ratio goes to infinity and the secondary masses have zero initial conditions, the system approaches the Newton's cradle model with precompression, a problem with a purely optical dispersion relation. In this case, traveling bright breathers were investigated in [10] via the analysis of the corresponding *focusing* NLS, which admits sech-type solutions.

To explore the weakly nonlinear dynamics at finite mass ratio, we use a multiscale asymptotic method (see [31,36,37] and references therein) and derive the classical NLS equation, yielding closed-form solutions of sech-type and tanh-type in the focusing and defocusing cases, respectively. In particular, we show that parameters (mass ratio and precompression) can modify the number of focusing regions in the acoustic and optical bands, a phenomenon which does not occur in classical homogeneous and diatomic granular chains [7,14]. This property is particularly interesting for applications because precompression is easy to tune experimentally. Another special feature of the resonant granular chain is that the cubic NLS coefficient vanishes at the zero wavenumber corresponding to the lower edge of the optical band. Since the NLS equation is defocusing at the upper edges of the optical and acoustic bands, the NLS equation cannot be used to approximate stationary bright breathers in the present context.

Having identified focusing and defocusing parameter regimes, we first investigate how well the focusing NLS equation approximates moving bright breather solutions of the original system. Provided that certain resonances are avoided, we find that the focusing NLS equation successfully approximates small-amplitude optical bright breathers at various mass ratios and wave numbers. This very good correspondence is established by integrating the lattice differential equation starting from the NLS approximation, which leads to robust motion of the bright breather over long times. We also demonstrate that bright breathers can be generated in the resonant granular chain initially at rest, and driven from a boundary at a frequency within the focusing region of the optical band (see [6,17] for related works). In addition, we analyze discrepancies between numerical solutions and NLS approximations which can be observed at some other wave numbers for the optical branch and in the acoustic case. In particular, in some cases we observe formation and robust propagation of *nanoptera*, bright breathers that emit small-amplitude oscillations behind them.

Following the approach in [7], we also consider the defocusing NLS at the edges of both optical and acoustic branches that correspond to wave number equal to π, and use the solutions of the modulation equation to construct the approximate standing dark breather solutions. A continuation procedure based on a Newton-type method with initial conditions built from the approximation ansatz is employed to compute numerically exact stationary dark breathers for a wide range of frequencies and at different mass ratios. Interestingly, the resulting branches of solutions also include large-amplitude dark breathers, whose dynamics is *strongly nonlinear*.

We examine numerically the stability of the exact dark breathers of both weakly and strongly nonlinear types, using both a Floquet analysis and direct numerical simulations. Our results suggest that small-amplitude weakly nonlinear dark breather solutions with frequencies close to the linear frequencies of the system are stable, in analogy to what was found for the homogeneous granular chain in [7]. As the amplitude of the dark breather solution becomes relatively large compared to the amount of precompression, the solution starts to exhibit a very strong modulational instability of the background, eventually leading to its complete destruction and the emergence of chaotic dynamics. However, when a real instability of the solution is dominant in the Floquet spectrum, it may give rise to steady propagation of a dark breather at large enough time. Interestingly, we observe that such types of propagating dark breathers can also form spontaneously, as a result of the instability of certain acoustic periodic traveling waves. We also show that the mass ratio plays a substantial role in oscillatory instabilities of the background of the dark breathers. In contrast, the value of the mass ratio has a less significant effect on real instability modes for both the strongly and weakly nonlinear solutions.

The paper is organized as follows. Section 2 introduces the model, and the dispersion relation for plane waves is derived. In Section 3 we derive the modulation equation of NLS type (with more technical details included in the Appendix), recall basic features of the focusing and defocusing regimes of NLS, and localize these different regimes in the parameter space of the original lattice. In Section 4 we investigate the existence of moving bright breathers for the original system at different mass ratios and test the validity of the NLS approximation. In Section 5 we analyze the existence and stability of stationary dark breathers and discuss the excitation of traveling dark breathers by different means. Concluding remarks can be found in Section 6.