Physica D 331 (2016) 81–88

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Suppression of Fermi acceleration in composite particles

Kellen Manoela Siqueira *, Marcus Aloizio Martinez de Aguiar

Instituto de Física 'Gleb Wataghin', Universidade Estadual de Campinas, Unicamp, 13083-970, Campinas, SP, Brazil

HIGHLIGHTS

- We study the motion of a composite particle in a one dimensional billiard.
- If the billiard walls are fixed the motion is chaotic for low energies.
- If the walls move periodically Fermi Acceleration may occur.
- The internal degrees of freedom slow down the particle.
- The composite particle gains energy at a smaller rate than a point particle.

ARTICLE INFO

Article history:
Received 13 October 2015
Received in revised form
17 May 2016
Accepted 18 May 2016
Available online 25 May 2016
Communicated by A. Pikovsky

Keywords:
Billiards
Fermi acceleration
Chaos
Hamiltonian mechanics

ABSTRACT

We study the motion of a composite particle in a one-dimensional billiard with a moving wall. The particle is modeled by two point masses coupled by a harmonic spring. We show that the energy gained by the composite particle is greatly reduced with respect to a single point particle. We show that the amount of energy transferred to the system at each collision with the walls is independent of the spring constant. However, the presence of the spring is responsible for the energy suppression because it diminishes the number of collisions by storing part of the system's energy and reducing the velocity of the particle's center of mass.

1. Introduction

Billiards with moving boundaries have been studied in the context of physics since 1949. In this year Fermi presented a model that was later modified by Ulam and that came to be known as "Fermi–Ulam model" (or FU model). The model was originally proposed as a way of explaining the high energy with which some charged particles of cosmic rays arrived in Earth. Fermi believed that these particles could accelerate as a result of successive collisions with structures in a moving magnetic field [1]. Ulam proposed a mechanical analogue of this model, the FU model, in which a particle moves between two walls of infinite mass, one fixed and one moving according to some rule. The particle gains energy in head on collisions and loses it in overtaking ones [2,3]. According to Fermi hypothesis if the oscillations of the wall were random one would expect that on average the particle would gain more energy than it lost and, therefore, it would accelerate. This phenomenon in which a particle gains unlimited amounts of energy due to collisions with a moving wall is called Fermi Acceleration (FA) [1,4].

Later work showed that the Fermi hypothesis was correct only when the movement of the wall was random. Lieberman e Lichtenberg [5,6], showed that if the motion of the wall is periodic in time the FU model could be interpreted as an autonomous Hamiltonian system with two degrees of freedom. In this case, if the motion of the wall is sufficiently smooth, KAM theory can be applied and due to KAM curves the energy of the system remains bounded and Fermi acceleration is not observed. Since then more realistic models were presented to explain the behavior of cosmic rays. But the study of billiards with moving walls continued in the context of dynamical systems and it has become a field of its own [7].

Billiards are simple systems that can have unexpectedly complex behavior. Even when the walls are not moving there is a wide range of possible dynamics that can be found in two or higher dimensional billiards. Depending on the shape of the billiard domain the dynamics can be regular, for integrable billiards, it can have a mixed phase or it can be fully chaotic [8–10]. When the walls are put into motion there is, in addition, the possibility of adding or removing energy from the system. For billiards with two or higher dimensions the average energy gained (or lost) depends on the shape

* Corresponding author. Tel.: +55 13997319302.
E-mail address: kellenms@ifi.unicamp.br (K.M. Siqueira).

http://dx.doi.org/10.1016/j.physd.2016.05.009

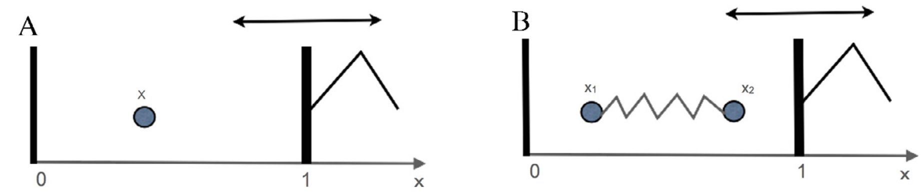

Fig. 1. Schematic illustration of the (A) FU model; (B) our model.

of the boundary and on the protocol used for the wall motion. Depending on the billiard it is possible to find protocols where the billiard ball gains energy indefinitely, that is, FA is achieved [8].

For two dimensional billiards a conjecture by Loskutov, Ryabov and Akinshin (LRA conjecture) states that if the static billiard has a chaotic phase space then FA can be achieved with moving walls [11]. This conjecture has been shown correct in the annular and stadium billiards [12,13]. However it has also been shown integrable billiards, such as the ellipse, can lead to FA for certain protocols of wall movement [14].

An important question regarding billiards that has not received much attention is how a particle's internal degrees of freedom would affect its behavior in a billiard with moving walls. Internal degrees of freedom often couple with the center of mass during collisions and the consequences of this coupling can be observed even in double slit experiments in quantum mechanics [15]. However, to the best of our knowledge, no work has yet explored the influence of the internal degrees of freedom of a composite particle in the phenomenon of Fermi acceleration.

The purpose of this work is to evaluate how a particle with internal degrees of freedom behaves in a billiard with moving walls. For simplicity we will consider a one-dimensional billiard with a single moving wall and model the composite particle as two point masses coupled by a harmonic spring. We will show that the particle's energy gain is significantly diminished with respect to the corresponding single particle and we will describe the mechanism that leads to that. In the next section we present the model used and in Section 3 we show and discuss the results of numerical simulations for billiards with fixed and moving walls.

2. Model

Our model consists of two point particles connected by a harmonic spring and moving in a one dimensional infinite well that has one moving boundary. The two particles and the spring can be seen as a single composite particle with internal degrees of freedom. The vibrational modes of the system can be excited when either particle collides with the walls.

The system is described by the Hamiltonian

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{1}{2}k(x_1 - x_2)^2 + V_{ext}^1(t) + V_{ext}^2(t) \tag{1}$$

where p_i, m_i and x_i are, respectively, the momentum, mass and position of the particle i. The two point particles interact via a harmonic potential with elastic constant k. The equilibrium distance between these particles is zero and they collide elastically at zero length.

V_{ext}^i describes the environment in which the particles are allowed to move.

$$V_{ext}^i(t) = \begin{cases} 0, & 0 \le x_i \le X_w(t) \\ \infty, & \text{otherwise} \end{cases} \tag{2}$$

as illustrated in Fig. 1.

We assume that the collisions between the particles and between each particle and the walls are elastic. We also assume that one particle cannot go through the other. This restricts the movement of the two particles to the interval $0 \le x_1 \le x_2 \le X_w(t)$. Therefore particle 1 collides only with the fixed wall and particle 2 with the moving wall.

The collision between particle 1 and the fixed wall only changes the direction of the velocity of this particle, $v_1 \longrightarrow -v_1$. The collision between particle 2 and the moving boundary is a perfect reflection if seen in the frame of the moving wall. Therefore,

$$v_2 \longrightarrow -v_2 + 2V_w(t). \tag{3}$$

Finally, the collision between particles 1 and 2 is elastic. Therefore, the particle velocities before (u_i) and after (v_i) the collision must satisfy the relations:

$$\begin{cases} m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \\ \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2. \end{cases} \tag{4}$$

By introducing the variables $\bar{x}_i = \sqrt{m_i}x_i$, $\bar{u}_i = \sqrt{m_i}u_i$ and $\bar{v}_i = \sqrt{m_i}v_i$ we can interpret this model in an alternative way. Instead of two particles moving in one dimension, we can think of the system as a single particle moving in a two dimensional space. In these variables the system's configuration space becomes a triangle (see Fig. 2) where the diagonal is given by $\bar{x}_1/\sqrt{m_1} = \bar{x}_2/\sqrt{m_2}$. In the new variables the reflection rules between the particles can be written as [16]:

$$\begin{cases} \sqrt{m_1}\bar{u}_1 + \sqrt{m_2}\bar{u}_2 = \sqrt{m_1}\bar{v}_1 + \sqrt{m_2}\bar{v}_2 \\ \bar{u}_1{}^2 + \bar{u}_2{}^2 = \bar{v}_1{}^2 + \bar{v}_2{}^2. \end{cases} \tag{5}$$

In this representation the single particle moves inside the triangle and undergoes elastic collisions at its boundaries. The spring between the two particles is now interpreted as an external potential for this one particle. The movement of the right wall corresponds to the movement of the top side of the triangle.

The region near the acute angles of the triangle can be a source of numerical errors when the equations of motion are solved numerically. In order to avoid this problem we use a geometrical tool known as unfolding. Since the collision between the particle and the diagonal of the triangle is elastic, and therefore the angle of incidence equals the angle of reflection, we can reflect the entire triangle instead of reflecting the particle (Fig. 2(B)). In the unfolded scheme, the movement of the wall is equivalent to the joint movement of the top and right sides of the square. The advantage here is that it is no longer necessary to consider the collisions between the particle and the diagonal (that is, the collision between the two original particles). Note that the diagonal of the original triangle is a trajectory corresponding to the particles moving together.

3. Results and discussion

3.1. Behavior for fixed walls

Previous studies of billiards with moving walls have shown that the existence, or non-existence, of Fermi acceleration depends strongly on the dynamical properties of the corresponding static billiard. In particular, numerical experiments show that particle acceleration is more likely to be observed in chaotic billiards [11].