Vol. 76 (2015) *REPORTS ON MATHEMATICAL PHYSICS* No. 2

COHERENT STATES FOR LANDAU LEVELS: ALGEBRAIC AND THERMODYNAMICAL PROPERTIES

ISIAKA AREMUA[1,2], MAHOUTON NORBERT HOUNKONNOU[2]
and EZINVI BALOÏTCHA[2]

[1]Université de Lomé (UL),
Faculté Des Sciences (FDS), Département de Physique, B.P. 1515 Lomé, Togo
[2]International Chair of Mathematical Physics and Applications,
ICMPA-UNESCO Chair, University of Abomey-Calavi, 072 B.P. 50 Cotonou, Republic of Benin
(e-mails: isiaka.aremua@cipma.uac.bj, norbert.hounkonnou@cipma.uac.bj, ezinvi.baloitcha@cipma.uac.bj

(*Received May 5, 2015 – Revised July 20, 2015*)

This work describes coherent states for a physical system governed by a Hamiltonian operator, in two-dimensional space, of spinless charged particles subject to a perpendicular magnetic field **B**, coupled with a harmonic potential. The underlying $\mathfrak{su}(1,1)$ Lie algebra and Barut–Girardello coherent states are constructed and discussed. Then, the Berezin–Klauder–Toeplitz quantization, also known as coherent state (or anti-Wick) quantization, is discussed. The thermodynamics of such a quantum gas system is elaborated and analyzed.

Keywords: isotropic harmonic potential, Landau levels, coherent states, quantization.

1. Introduction

The system of charged quantum particles interacting with a constant magnetic field is undoubtedly one of the most thoroughly investigated systems in quantum mechanics, mainly inspired by condensed matter physics and quantum optics. A family of coherent states (CS) adapted to such a system was first proposed in [1]. In [2], the behaviour of the transverse motion of electrons in an external uniform magnetic field **B** was considered. A complete set of CS wave packets was constructed. These states are non-spreading packets of minimum uncertainty that follow a classical motion. They are the eigenstates of two non-Hermitian operators that annihilate the (zero-) energy and angular momentum ground states, respectively. The CS basis was used for the calculation of the partition function. Landau diamagnetism and de Haas–van Alphen oscillations are contained in this setting.

Some alternative constructions were proposed in [3] and [4]. In metal and other dense electronic systems, the electrons occupy many Landau levels. Furthermore, the kinetic energy levels of electrons in two-dimensional gas correspond to Landau levels. In [5], the generalized Gazeau–Klauder CS were extended to systems with more than one degree of freedom. There, three different types of these generalized CS were considered. The $\mathfrak{su}(2)$ and $\mathfrak{su}(1,1)$ symmetries of isotopic harmonic oscillator in two-dimensional spinless charged particles in the presence of a constant magnetic field were investigated. CS also play an important role in nonequilibrium statistical physics. They describe the evolution towards thermodynamic equilibrium for quantum systems with equidistant energy spectra [6].

CS were also investigated to obtain Landau diamagnetism for a free electron gas [7]. In [8] the generalized Klauder–Perelomov [6] and Gazeau–Klauder [9] CS of Landau levels were constructed using two different representations for the Lie algebra $\mathfrak{h}_4$. In [10], the Landau levels were reorganized into two different hidden symmetries, namely $\mathfrak{su}(2)$ and $\mathfrak{su}(1,1)$. The representation of $\mathfrak{su}(1,1)$ by the Landau levels then led to the construction of the Barut–Girardello CS (BGCS) [11]. In [12], the Glauber two-variable CS were developed in various representations using a unitary displacement operator. The Klauder–Perelomov CS of $\mathfrak{su}(1,1)$ and $\mathfrak{su}(2)$ algebras, for Landau levels, minimizing the Heisenberg uncertainty relation, and their statistical properties were discussed. More recently, Bergeron et al. [13] investigated the consistency of CS quantization (also named Berezin–Klauder–Toeplitz or anti-Wick quantization), and reached the conclusion that the predictions resulting from this type of quantization and those resulting from canonical quantization are compatible on a physical level for nonrelativistic systems, even though these two quantization techniques are not mathematically equivalent.

This work aims at considering Landau levels for a Hamiltonian operator describing the motion, in two-dimensional space, of spinless charged particles subject to a perpendicular magnetic field **B** coupled with a harmonic potential. The underlying $\mathfrak{su}(1,1)$ Lie algebra and BGCS are constructed and discussed. Then, the Berezin–Klauder–Toeplitz quantization is performed. The thermodynamics of such a quantum gas system is elaborated and analyzed.

The paper is organized as follows. In Section 2, we start with the study of a spinless charged particles gas on the (x, y)-space in a magnetic field **B** with an isotropic harmonic potential. We use step and orbit-center coordinate operators. In Section 3, the $\mathfrak{su}(1,1)$ representation in the Hamiltonian quantum states is studied. There follows a discussion of the BGCS of the Lie algebra $\mathfrak{su}(1,1)$. The mean values of $\mathrm{SU}(1,1)$ group generators and of the physical system observables, the probability density and the time dependence of the BGCS are calculated. The quantization of a complex plane using these states is investigated in Section 4. The Mandel parameter is studied in Section 5. In Section 6, the main statistical properties for the quantum gas in thermodynamic equilibrium with a reservoir at temperature T are computed and analyzed. Section 7 is a summary.

2. Hamiltonian operator of an electron in a uniform magnetic field with a harmonic potential

Consider a system of spinless charged particles confined to two-dimensional (x, y)-space, with a magnetic field **B** applied along the z-direction. The eigenstates and eigenvalues of such a system were investigated for the first time by Landau [7]. When a harmonic potential is introduced and the Coulomb interactions are neglected,