# CONSTRUCTION METHOD OF ANALYTICAL SOLUTIONS TO THE MATHEMATICAL PHYSICS BOUNDARY PROBLEMS FOR NON-CANONICAL DOMAINS

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The majority of practical cases of acoustics problems requires solving the boundary problems in non-canonical domains. Therefore construction of analytical solutions of mathematical physics boundary problems for non-canonical domains is both lucrative from the academic viewpoint, and very instrumental for elaboration of efficient algorithms of quantitative estimation of the field characteristics under study. One of the main solving ideologies for such problems is based on the superposition method that allows one to analyze a wide class of specific problems with domains which can be constructed as the union of canonically-shaped subdomains. It is also assumed that an analytical solution (or quasi-solution) can be constructed for each subdomain in one form or another. However, this case implies some difficulties in the construction of calculation algorithms, insofar as the boundary conditions are incompletely defined in the intervals, where the functions appearing in the general solution are orthogonal to each other. We discuss several typical examples of problems with such difficulties, we study their nature and identify the optimal methods to overcome them.
**Keywords:** method of partial subdomains, superposition method, potential theory, non-canonical domain, sonic emission, intersected cylinders.

## 1. Introduction

Analytical solutions for various boundary conditions of mathematical physics have played a vital role in the development of this field of physics. At present the most full description of the available procedures of deriving such solutions and systematisation of the accumulated results is presented in the handbook of Morse and Feshbach [1].

From the standpoint of solving applied problems, analytical solutions have only one deficiency – the number of such solutions is quite limited. For this reason, the elaboration of approaches that expanded the scope of domains, for which effective and instrumental solutions could be constructed for quantitative assessment of physical field characteristics, has been under way for the last decades. Such elaboration included e.g. the so-called Schwarz algorithm [2], the T-matrix method [3–6], the Shestopalov method [7] of solving the Riemann–Hilbert problem, etc. Such approaches are based on subdivision of complex-shaped domains into relatively simple subdomains, for which analytical representations of the derived functions can be algebraic equations, whose properties allow the reduction method to be applied. In some cases, such as applications of the T-matrix method, the derived solutions are valid only within a limited range of the domain geometry variation.

The analysis of available works in this field reveals that the potentialities of the method of domain decomposition [2, 8–10] are not yet exploited to the fullest practicable extent. This study presents a number of problems, derivation of solutions for which illustrates the new possibilities of constructing analytical solutions for complex-shaped domains. In particular, some problems related to the Laplace and Helmholtz equations are discussed. Application of the alternative concept of general solution opens new horizons for wide applications of the proposed approach to solve a wide range of problems of mathematical physics.

## 2. Main features of the method of partial subdomains

Consider an elementary problem on sound propagation in the composite waveguide filled with an ideal liquid of density $\rho_0$ and a sound velocity $c_0$. The problem geometry is depicted in Fig. 1. For the sake of definiteness, the waveguide walls are considered to be acoustically rigid. All characteristics of the harmonic sound field within this domain can be derived via the function of potential of velocities $\Phi$, which satisfies the Helmholtz equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + k^2\Phi = 0, \qquad k = \frac{\omega}{c_0}. \tag{1}$$

Here $\omega$ is the frequency of a harmonic wave, while the time factor is set in the form of $\exp(-i\omega t)$.

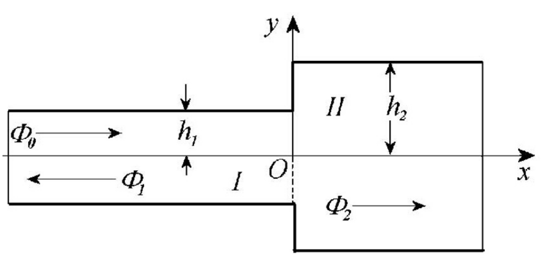


**Fig. 1.** Geometry of a composite waveguide.