NEW GENERALIZED COHERENT STATES ARISING FROM GENERATING FUNCTIONS: A NOVEL APPROACH

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(*Received August 14, 2014 – Revised October 18, 2014*)

We introduce in this paper new kinds of coherent states for some quantum solvable models such as an electron moving in flat surface subject to perpendicular constant and decaying (Morse like) magnetic field. We explain how these states come directly from generating functions of certain families of classical orthogonal polynomials without the complexity of algebraic approaches. It is shown that these states realize resolution of the identity property through some positive-definite measures.

Keywords: classical polynomials, generating functions, Landau levels, quantum solvable models, algebraic methods in quantum mechanics.

1. Introduction

1.1. Review and motivations

According to the pioneering work of Schrödinger [1], coherent states in their general form can be realized as Gaussian wave function. It can be constructed from a particular superposition of wave functions corresponding to discrete eigenvalues of harmonic oscillator. They play important roles in quantum optics and provide us with a link between quantum and classical oscillators. Here, it is necessary to emphasize that quantum coherence of states nowadays pervades many branches of physics such as quantum electrodynamics, solid-state physics, and nuclear and atomic physics from both theoretical and experimental points of view. Successfully, these states were applied to some other models considering their Lie algebra symmetries by Glauber [2], Klauder [3], Sudarshan [4], Barut and Girardello [5] and Perelomov [6]. Additionally, for the models with one degree of freedom either discrete or continuous spectra—with no remark on the existence of a Lie algebra symmetry—Gazeau et al. proposed new coherent states, which were parametrized by two real parameters [7]. Also, there exist some considerations in connection with coherent states corresponding to the shape invariance symmetries [8]. To construct coherent states, four main different approaches of the so-called Schrödinger, Klauder–Perelomov (K–P), Barut–Girardello (B–G) and Gazeau–Klauder (G–K) methods have been found, so that the second and the third approaches rely directly on the Lie algebra symmetries and their corresponding generators. Clearly they introduce coherent states as superpositions of Hamilton's eigenvectors which span complete and (bi-)orthogonal Hilbert spaces [9].

Based on the fact that coherent states may usually be led to the generating functions of some (bi-)orthogonal polynomials [10], we are motivated here to follow the reverse idea. In other words we establish new kinds of coherent states through the application of generating functions. We therefore review some of them in detail as follows.

1.2. Some trivial examples

1.2.1. Coherent states arising from the generating function of classical Hermite polynomials

Using the Hermite polynomials $H_n(x)$ [11]

$$G(x,t) = e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x), \qquad |t| < \infty, \tag{1}$$

with the substitution $t\sqrt{2} \to z \in C$ one gets

$$\begin{aligned} G(x,z) = e^{xz\sqrt{2}-\frac{z^2}{2}} &= \pi^{1/4} e^{\frac{x^2}{2}} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} \phi_n(x) \\ &= \pi^{1/4} e^{\frac{x^2+|z|^2}{2}} \langle x \mid z \rangle_{\mathrm{Sch}}, \end{aligned} \tag{2}$$

where by $\phi_n(x)\Big(= \frac{e^{-\frac{x^2}{2}}}{\sqrt{2^n n! \sqrt{\pi}}} H_n(x)\Big)$ we denote the complete and orthonormal set of eigenvectors of the simple harmonic oscillator (SHO), i.e.

$$\int_{-\infty}^{\infty} \phi_n(x)\phi_m(x)dx = \delta_{nm},$$

and by $\langle x \mid z \rangle_{\mathrm{Sch}}\Big(= e^{-\frac{|z|^2}{2}} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} \phi_n(x)\Big)$ we denote the Schrödinger type of coherent states[1] attached to it, respectively. Clearly, Eq. (2) indicates that using generating function of the classical orthogonal polynomials one can easily find[2] the coherency of quantum states, i.e.

$$\langle x \mid z \rangle_{\mathrm{Sch}} = \frac{e^{-\frac{x^2+|z|^2}{2}}}{\pi^{1/4}} G(x,z). \tag{3}$$

[1] The subscript 'Sch' refers to Schrödinger and specifies a particular type of quantum states that are called the canonical coherent states, too.

[2] However, as will be discussed later, this cannot be applied to all of the classical orthogonal polynomials.